EQUIVARIANT PROJECTION MORPHISMS OF SPECHT MODULES

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Let d be a positive integer, and let $\mathfrak{S}_d$ denote the symmetric group on d letters. Given a partition λ of d, the Specht module V_λ is a finite dimensional vector space over $\mathbb{C}$ which admits a natural basis indexed by all standard tableaux of shape λ with entries in $\{1, 2, \ldots, d\}$. It affords an irreducible representation of the symmetric group $\mathfrak{S}_d$, and conversely every irreducible representation of $\mathfrak{S}_d$ is isomorphic to V_λ for some partition λ.

Given two Specht modules V_λ, V_μ, their tensor product representation $V_\lambda \otimes V_\mu$ is in general reducible, and hence it splits into a direct sum $\bigoplus\limits_{\nu} V_\nu^{m_\nu}$ of irreducibles. This raises the problem of describing the $\mathfrak{S}_d$-equivariant projection morphisms (alternately called $\mathfrak{S}_d$-homomorphisms) of the form $V_\lambda \otimes V_\mu \longrightarrow V_\nu$ in terms of the standard tableaux bases. In this work we give explicit formulae describing this morphism in the following cases:

- $V_{(d-1,1)} \otimes V_{(d-1,1)} \longrightarrow V_{(d-1,1)}$,
- $V_{(1^d)} \otimes V_\lambda \longrightarrow V_{\tilde{\lambda}}$, where $\lambda = (d-1, 1)$ or $(2, 1^{d-2})$ and $\tilde{\lambda}$ is its conjugate.

The isomorphism $V_\lambda \simeq V_\lambda^*$ induces an equivariant projection morphism (which we call a q-morphism)

$$V_\lambda \otimes V_\lambda \longrightarrow V_{(d)} \cong \mathbb{C}.$$

We have found explicit formulae for this morphism in the following cases:

$$\lambda = (d-1, 1),\ (d-2, 1, 1),\ (2, 1, \ldots, 1).$$

Finally, we present a conjectural formula for the q-morphism in the case

$$\lambda = (d-r, 1, \ldots, 1).$$