Volumes

MATH 1700

Readings

Readings: Section 6.2, 6.3

Volumes of some simple shapes

• The volume of a rectangle of length l, width w and height h is $V = lwh$.

• A **cylinder** in math is any shape obtained by translating a planar region perpendicular to itself a distance h. The volume of a cylinder is the area A of the region times h: $V = Ah$.

Definition of Volume

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Let S be a solid which lies between $x = a$ and $x = b$. Assume that the area of the slice at x is $A(x)$, and that $A(x)$ is a continuous function of x. Then

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If the shape S lies between $y = a$ and $y = b$ and the slice at y has area $A(y)$, then

$$\text{Volume} = \int_a^b A(y)dy.$$

Special case: "washer method" for solids of revolution

Rotated around a line parallel to the x-axis:
Let R be a region between $x = a$ and $x = b$.
The volume of the region obtained by rotating R around a line parallel to the x-axis is

$$V = \int_a^b A(x)dx = \int_a^b \left(\pi(\text{outer radius})^2 - \pi(\text{inner radius})^2\right) dx.$$

Note: you must write the inner and outer radius as a function of x.

Special case: "washer method" for solids of revolution

Rotated around a line parallel to the y-axis:
Let R be a region between $y = c$ and $y = d$.
The volume of the region obtained by rotating R around a line parallel to the y-axis is

$$V = \int_c^d A(y)dy = \int_c^d \left(\pi(\text{outer radius})^2 - \pi(\text{inner radius})^2\right) dy.$$

Note: you must write the inner and outer radius as a function of y.

The volume of a thick cylinder

Let S be a cylinder of outer radius r_2, inner radius r_1 and height h. Its volume is

$$V = \pi r_2^2 h - \pi r_1^2 h = \pi(r_2 + r_1)(r_2 - r_1)h.$$

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If we set $r = (r_1 + r_2)/2$ and $\Delta r = r_2 - r_1$ then

$$\Delta V = 2\pi r h \Delta r.$$

Special case: cylindrical shell method for solids of revolution

Let R be the region under $y = f(x)$ and above the x axis between $x = a$ and $x = b$.

Let S be the solid obtained by rotating R about the y-axis.

Then

$$V = \int_a^b 2\pi x f(x) dx.$$

Special case: cylindrical shell method for solids of revolution

In general,
let R be a region in the x/y plane between $x = a$ and $x = b$
and S be the solid obtained by rotating R about the y-axis.
If $h(x)$ is the height of the cylindrical shell as a function of x, then

$$V = \int_a^b 2\pi x h(x) dx.$$

General warning

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Draw pictures!