Trigonometric integrals

MATH 1700

Readings

Readings: Section 7.2

One odd power in $\int \sin^m x \cos^n x$

- If the power of $\cos x$ is odd, use $\cos^2 x = 1 - \sin^2 x$ to eliminate all but one $\cos x$:

$$\int \sin^m x \cos^{2k+1} x = \int \sin^m x (1 - \sin^2 x)^k \cos x dx.$$

Substitute $u = \sin x$.

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$$\int \sin^{2k+1} x \cos^n x dx = \int \sin x (1 - \cos^2 x)^k \cos^n x dx.$$

Substitute $u = \cos x$.

Both powers even in $\int \sin^m x \cos^n x$

If both powers of $\sin x$ and $\cos x$ are both even, use the double-angle formulas

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x) \quad \text{or} \quad \cos^2 x = \frac{1}{2}(1 + \cos 2x).$$

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Sometimes this double-angle formula is also useful:

$$\sin 2x = 2 \sin x \cos x.$$

$\int \tan^m x \sec^n x dx$

• If the power of $\sec x$ is even, use $\sec^2 x = 1 + \tan^2 x$ to eliminate all but $\sec^2 x$:

$$\int \tan^m x \sec^{2k} x dx = \int \tan^m x (1 + \tan^2 x)^{k-1} \sec^2 x dx.$$

Substitute $u = \tan x$.

$\int \tan^m x \sec^n x dx$

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$$\int \tan^m x \sec^{2k} x dx = \int \tan^m x (1 + \tan^2 x)^{k-1} \sec^2 x dx.$$

Substitute $u = \tan x$.

• If the power of $\tan x$ is odd, use $\tan^2 x = \sec^2 x - 1$ to write everything in terms of $\sec x$ except for one $\sec x \tan x$:

$$\int \tan^{2k+1} x \sec^n x dx = \int (\sec^2 x - 1)^k \sec^{n-1} x \sec x \tan x.$$

Substitute $u = \sec x$.

$\int \tan^m x \sec^n x dx$

In other cases, these tricks sometimes help:

$$\int \tan x dx = \ln|\sec x| + C$$

$$\int \sec x dx = \ln|\sec x + \tan x| + C.$$

$\int \sin mx \cos nx$, $\int \cos mx \cos nx$, $\int \sin mx \sin nx$

To evaluate any of the integrals

$$\int \sin mx \cos nx, \quad \int \cos mx \cos nx, \quad \int \sin mx \sin nx$$

use one of these identities:

$$\sin A \cos B = \frac{1}{2}\left(\sin(A-B) + \sin(A+B)\right)$$
$$\sin A \sin B = \frac{1}{2}\left(\cos(A-B) - \cos(A+B)\right)$$
$$\cos A \cos B = \frac{1}{2}\left(\cos(A-B) + \cos(A+B)\right).$$