Trigonometric substitution

MATH 1700

Readings

Readings: Section 7.3

An inverse substitution is a substitution of the form

$$x = g(t), \quad dx = g'(t)dt.$$

$$\int f(x)\,dx = \int f(g(t))g'(t)\,dt.$$

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How can you tell before you integrate? There are patterns to look for.

if you see $\sqrt{a^2 - x^2}$ try $x = a \sin\theta, \ -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

and use the identity $1 - \sin^2\theta = \cos^2\theta$.

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if you see $\sqrt{a^2 + x^2}$ try $x = a\tan\theta, \; -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

and use the identity $1 + \tan^2\theta = \sec^2\theta$.

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if you see $\sqrt{x^2 - a^2}$ try $x = a\sec\theta,\ 0 \le \theta < \frac{\pi}{2}$ or $\pi \le \theta < \frac{3\pi}{2}$

and use the identity $\sec^2\theta - 1 = \tan^2\theta$.