Surface Area

MATH 1700

Readings

Readings: Section 8.2

Surface of revolution around x-axis: if $y = f(x)$

Let $y = f(x)$ be a function on an interval $[a, b]$, such that f' is continuous on $[a, b]$.

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The **surface area** of the surface obtained by rotating the graph around the x-axis is

$$\begin{aligned} S &= \int_a^b 2\pi f(x)\sqrt{1 + f'(x)^2}dx \\ &= \int_a^b 2\pi y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}dx \\ &= \int_a^b 2\pi y\, ds. \end{aligned}$$

Surface of revolution around x-axis: if $x = g(y)$

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$$S = \int_c^d 2\pi y \sqrt{1 + g'(y)^2} dy$$

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$$= \int_c^d 2\pi y \, ds.$$

The *ds* formula

It's a bit easier to remember the surface area formula, if you can remember

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx = \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy.$$

and insert it in

$$\int_{lower\ bound}^{upper\ bound} 2\pi y\, ds;$$

if you integrate with respect to y, use the y bounds $[c, d]$; if you integrate with respect to x, use the x bounds $[a, b]$.

Surface of revolution about the y-axis: if $y = f(x)$

Let $y = f(x)$ be a function on $[a, b]$ such that f' is continuous on $[a, b]$. The surface area of the surface obtained by rotating the graph of f around the y-axis is

$$\begin{aligned} S &= \int_a^b 2\pi x \, ds \\ &= \int_a^b 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \\ &= \int_a^b 2\pi x \sqrt{1 + f'(x)^2} dx. \end{aligned}$$

Surface of revolution about the y-axis: if $x = g(y)$

Let $x = g(y)$ be a function on $[c, d]$ such that g' is continuous on $[c, d]$. The surface area of the surface obtained by rotating the graph of f around the y-axis is

$$\begin{aligned} S &= \int_c^d 2\pi x\, ds \\ &= \int_c^d 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy \\ &= \int_c^d 2\pi g(y) \sqrt{1 + g'(y)^2} dy. \end{aligned}$$

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(1) decide whether to use $\int 2\pi x\, ds$ or $\int 2\pi y\, ds$ based on geometry:
 (a) if rotated around the y-axis, use $2\pi x\, ds$
 (b) if rotated around the x-axis, use $2\pi y\, ds$.

(2) in the integrand, you have a choice of
 (a) integrating with respect to x, or
 (b) integrating with respect to y,

depending on what is convenient for the problem. In (a) you must write the integrand in terms of x and in (b) you must write the integrand in terms of y.

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- (b) if rotated around the x-axis, use $2\pi y\, ds$.

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depending on what is convenient for the problem. In (a) you must write the integrand in terms of x and in (b) you must write the integrand in terms of y.

Note: If you make the wrong choice for (1), you will be completely wrong. For (2), both choices are correct in principle, but one of the choices might make the problem much more complicated.