The Substitution Rule

MATH 1700

Readings

Readings: Section 5.5

The substitution rule for indefinite integrals

Theorem

Let $u = g(x)$ be a differentiable function whose range is an interval I and let f be a continuous function on I. Then

$$\int f(g(x))g'(x)dx = \int f(u)du.$$

The substitution rule for definite integrals

Theorem

Assume that g' is continuous on $[a, b]$ and f is continuous on the range of g. Then

$$\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du.$$

Integrals of symmetric functions

Assume that f is continuous on $[-a, a]$.

- If f is an even function on $[-a, a]$ then

$$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx.$$

- If f is an odd function on $[-a, a]$ then

$$\int_{-a}^{a} f(x)dx = 0.$$