Indefinite Integrals and the Net Change Theorem

MATH 1700

Readings

Readings: Section 5.4

What is an indefinite integral?

An **indefinite integral** of a function f is an anti-derivative of f, and is denoted $\int f(x)dx$. That is:

$$\int f(x)dx = F(x) \quad \textbf{means} \quad F'(x) = f(x)$$

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Convention #2: The formula for an anti-derivative is always expressed on a single interval.

Table of indefinite integrals

1. $\int cf(x)dx = c\int f(x)dx$
2. $\int (f(x)dx + g(x)dx) = \int f(x)dx + \int g(x)dx$
3. $\int kdx = kx + C$
4. $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ for $n \neq -1$
5. $\int \frac{1}{x}dx = \ln|x| + C$
6. $\int e^x dx = e^x + C$
7. $\int a^x dx = \frac{a^x}{\ln a} + C$

Table of more indefinite integrals

8. $\int \sin x dx = -\cos x + C$
9. $\int \cos x dx = \sin x + C$
10. $\int \sec^2 x dx = \tan x + C$
11. $\int \csc^2 x dx = -\cot x + C$
12. $\int \sec x \tan x dx = \sec x + C$
13. $\int \csc x \cot x dx = -\csc x + C$

Table of still more indefinite integrals

14. $\displaystyle\int \frac{1}{x^2+1}\,dx = \tan^{-1} x + C$

15. $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$

16. $\displaystyle\int \sinh x\,dx = \cosh x + C$

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The Net Change Theorem

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This isn't really a new theorem, it's just the Fundamental Theorem of Calculus Part II stated differently.