The Fundamental Theorem of Calculus

MATH 1700

Readings

Readings: Section 5.3

Fundamental Theorem of Calculus, part I

Theorem

If f is continuous on $[a, b]$, then

$$g(x) = \int_a^x f(t)dt$$

is continuous on $[a, b]$ and differentiable on (a, b), and

$$g'(x) = f(x).$$

Fundamental Theorem of Calculus, part II

Theorem

If f is continuous on $[a, b]$, and $F'(x) = f(x)$ on $[a, b]$, then

$$\int_a^b f(t)dt = F(b) - F(a).$$

Terminology: If $F' = f$ then F is called an **anti-derivative** of f.

Important distinction: “Anti-derivative” does **not** mean “integral”. The FTC tells us that they are related.

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Important facts:

1. if F is an anti-derivative of f on an interval I, then $G(x) = F(x) + c$ is an anti-derivative of f on I.
2. so there are many anti-derivatives for any one function f.
3. If F and G are both anti-derivatives of f on an interval I, $G(x) - F(x) = c$ for some constant c.

Application of the fundamental theorem of calculus part II

If you can find an anti-derivative, then you can use the fundamental theorem part II to find definite integrals.