ex Show that

$$|x-2|<2 \Rightarrow \frac{1}{|x-7|} < \frac{1}{3}$$

using the triangle inequality.

By Corollary 2.2.4(a),

$$|x|-2 \le |x-2| < 2$$

So $|x|<4$.

So again by Corollary 2.2.4(a)

$$|7|-|x| \le |7-x|$$

So since $7-|x| > 7-4 = 3$ (strict)

$3 \le |7-x|$. In particular $|7-x| > 0$

so $\frac{1}{|7-x|} \le \frac{1}{3}$ (by Theorem 2.1.7(e) twice).