MATH 2080 F18 Assignment 3

Due Date: Monday November 19th, in lecture

Important:

- Just working on the problem sets is insufficient. You should be doing plenty of exercises from the book and lecture on your own.
- The questions are taken from the fourth edition of Bartle and Sherbert, and the numbering has changed. If you have an earlier edition, please consult with me or with a classmate to make sure that you have the right question. *If you do the wrong question you will not receive credit.*

1. Let $X = (x_n)$ be a convergent sequence such that $x_n \geq 3$ for all $n \in \mathbb{N}$. Assume that
$$\lim \frac{x_n^2}{3x_n - 4} = 2.$$
Prove that $\lim X = 4$.

2. Let $(X) = (x_n)$ be the sequence defined recursively by $x_1 = 1$ and $x_{n+1} = \sqrt{x_n + 12}$ for all $n \in \mathbb{N}$ such that $n \geq 1$.
   (a) Use induction to show that the sequence is bounded above by 4.
   (b) Prove that the limit exists.
   (c) Evaluate the limit (with a proof).

3. Section 3.3 # 9. (third edition, same number).

4. Consider the sequence
$$a_n = 1 + \cos(\pi n), \quad n \in \mathbb{N}.$$
Show that (a_n) does not converge. *Hint*: give a simple proof, using a theorem in Section 3.4.

5. Let $a_n = \sin(n^2)$ for $n \in \mathbb{N}$. Does (a_n) have a convergent subsequence? Prove it.

6. Consider the decimal number
$$x = .b_1 b_2 b_3 b_4 b_5 \cdots$$
and the associated sequence
$$a_n = .b_1 b_2 \cdots b_n.$$
   (a) For $m \geq n$, find an upper bound for $|a_m - a_n|$.
   (b) Prove that (a_n) is a Cauchy sequence. Do NOT mention the point x in your proof.