

# MATH 2080 F18 Assignment 1

Due Date: Friday, September 28th *in lecture*

## Important:

- Just working on the problem sets is insufficient. You should be doing plenty of exercises from the book and lecture on your own.
- The questions are taken from the fourth edition of Bartle and Sherbert, and the numbering has changed. If you have an earlier edition, please consult with me or a classmate to make sure that you have the correct question. *If you do the wrong question you will not receive credit.*

1. Let  $f : A \rightarrow B$  be a function.
  - (a) We say that  $g : B \rightarrow A$  is a “left inverse” for  $f$  if  $g \circ f(x) = x$  for all  $x \in A$ . Show that if  $f$  has a left inverse, then  $f$  is injective.
  - (b) We say that  $h : B \rightarrow A$  is a “right inverse” for  $f$  if  $f \circ h(x) = x$  for all  $x \in B$ . Show that if  $f$  has a right inverse, then  $f$  is surjective.
2. 1.1 #6 (a). (1.1 #4 (a) in 3rd edition).
3. Let  $f$  be the function with domain  $\{x \in \mathbb{R} : x \neq 2\}$  and co-domain  $\mathbb{R}$ , defined by  $f(x) = 3 + \frac{1}{(x-2)^2}$ .
  - (a) Let  $E = \{x : 4 < x\}$ . What is  $f(E)$ ? Give a proof.
  - (b) Let  $H = \{x : 7 \leq x \leq 12\}$ . What is  $f^{-1}(H)$ ? Give a proof.
4. Let  $S$  be the set of natural numbers which are evenly divisible by 7 and greater than 106. Show that  $S$  is denumerable.
5. State whether each one of the following claims is true or false. If the claim is true, prove it. If it is false, give a counterexample. (A counterexample is an example that shows that the claim is false).
  - (a) If  $S$  is a set,  $T \subseteq S$  and  $T \neq S$ , then there is no bijection between  $T$  and  $S$ .
  - (b) Whenever  $A_1 \subseteq \mathbb{N}$ ,  $A_2 \subseteq \mathbb{N}, \dots$  then  $\cup_{i=1}^{\infty} A_i$  is countable.
  - (c) If  $A_1, A_2, \dots$  are finite sets, then  $\cup_{i=1}^{\infty} A_i$  is denumerable.
6. 2.1 # 2 (b) (d). (3rd edition # 2 (b) (d)).
7. 2.1 # 5. (3rd edition # 5).