# Arc Length

MATH 1700

# Readings

**Readings**: Section 8.1

# Computing arc length of a graph

**Theorem**

*Let $f$ be a function such that $f'(x)$ is continuous on $[a, b]$. The length of the graph between $a$ and $b$ is*

$$L = \int_a^b \sqrt{1 + f'(x)^2}\, dx.$$

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**Variation**: if the function is $x = g(y)$ on $[c, d]$ then the length is $\int_c^d \sqrt{1 + g'(y)^2}dy$.

# Where does this come from?

We need a definition of length, in order to get the formula on the previous slide. The length can be approximated by straight lines:

$$L \cong \sum_{i=1}^{n} \sqrt{\Delta x_i^2 + \Delta y_i^2}$$
$$= \sum_{i=1}^{n} \sqrt{\Delta x_i^2 + f'(x_i^*)^2 \Delta x_i^2} \quad \text{(using MVT)}$$
$$= \sum_{i=1}^{n} \sqrt{1 + f'(x_i^*)^2} \Delta x_i.$$

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The actual length is defined to be the limit

$$L = \lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{\Delta x_i^2 + \Delta y_i^2} = \int_a^b \sqrt{1 + f'(x)^2} dx.$$

# Arc length function

**Definition**

Let $f$ be a function such that $f'$ is continuous on $[a, b]$. The arc length function on $[a, b]$ is

$$s(x) = \int_a^x \sqrt{1 + f'(t)^2}\,dt.$$

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**Idea**: it's the length between $a$ and $x$. It's a function because it depends on $x$.