Inverse trigonometric functions

MATH 1700

Readings

Readings:

• 7th edition: Section 1.6, "Inverse Trigonometric Functions", pp 67 - 69.

8th edition: Section 1.5, "Inverse Trigonometric Functions", pp 64 - 66.

• Both editions: Section 3.5, "Derivatives of Inverse Trigonometric Functions", pp 213 - 214.

Recall inverse function

Let $f : [a, b] \to [c, d]$ be an increasing (or decreasing) function. The **inverse** of f is the function f^{-1} is defined by:

Recall inverse function

Let $f : [a, b] \to [c, d]$ be an increasing (or decreasing) function. The **inverse** of f is the function f^{-1} is defined by:

$f^{-1}(x)$ is the unique number $y \in [a, b]$ such that $f(y) = x$.

Definition of inverse sine function

Definition

For $-1 \leq x \leq 1$, $\sin^{-1} x$ is the unique number y in $[-\pi/2, \pi/2]$ such that $\sin y = x$.

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So

$$\sin^{-1}(x) = y \Leftrightarrow \left(\sin y = x \text{ and } -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}\right).$$

and

$$\sin^{-1}(\sin x) = x \quad \text{whenever} \quad -\pi/2 \leq x \leq \pi/2$$
$$\sin(\sin^{-1} x) = x \quad \text{whenever} \quad -1 \leq x \leq 1.$$

Definition of inverse cosine function

Definition

For $-1 \leq x \leq 1$, $\cos^{-1} x$ is the unique number y in $[0, \pi]$ such that $\cos y = x$.

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For $-1 \leq x \leq 1$, $\cos^{-1} x$ is the unique number y in $[0, \pi]$ such that $\cos y = x$.

So

$$\cos^{-1}(x) = y \Leftrightarrow (\cos y = x \text{ and } 0 \leq y \leq \pi)\,.$$

and

$$\begin{aligned} \cos^{-1}(\cos x) &= x \quad \text{whenever} \quad 0 \leq x \leq \pi \\ \cos(\cos^{-1} x) &= x \quad \text{whenever} \quad -1 \leq x \leq 1. \end{aligned}$$

Definition of inverse tangent function

Definition

For any x, $\tan^{-1} x$ is the unique number y in $(-\pi/2, \pi/2)$ such that $\tan y = x$.

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So

$$\tan^{-1}(x) = y \Leftrightarrow \left(\tan y = x \text{ and } -\frac{\pi}{2} < y < \frac{\pi}{2}\right).$$

and

$$\tan^{-1}(\tan x) = x \quad \text{whenever} \quad -\pi/2 < x < \pi/2$$
$$\tan(\tan^{-1} x) = x \quad \text{whenever} \quad -\infty < x < \infty.$$

Limits of inverse tangent

We also have the following limits:

$$\lim_{x\to -\infty} \tan^{-1} x = -\frac{\pi}{2} \quad \text{and} \quad \lim_{x\to\infty} \tan^{-1} x = \frac{\pi}{2}.$$

These can be seen as the counterparts of the limits

$$\lim_{x\to -\frac{\pi}{2}^+} \tan x = -\infty \quad \text{and} \quad \lim_{x\to \frac{\pi}{2}^-} \tan x = \infty.$$

The other inverse functions

$$y = \csc^{-1} x \;\; x \in (-\infty, -1] \cup [1, \infty) \;\; \Leftrightarrow \csc y = x \;\; y \in (0, \pi/2] \cup (\pi, 3\pi/2]$$

$$y = \sec^{-1} x \;\; x \in (-\infty, -1] \cup [1, \infty) \;\; \Leftrightarrow \sec y = x \;\; y \in [0, \pi/2) \cup [\pi, 3\pi/2)$$

$$y = \cot^{-1} x \quad x \in (-\infty, \infty) \quad \Leftrightarrow \cot y = x \quad y \in (0, \pi).$$

The other inverse functions

$$y = \csc^{-1} x \;\; x \in (-\infty, -1] \cup [1, \infty) \;\; \Leftrightarrow \csc y = x \;\; y \in (0, \pi/2] \cup (\pi, 3\pi/2]$$

$$y = \sec^{-1} x \;\; x \in (-\infty, -1] \cup [1, \infty) \;\; \Leftrightarrow \sec y = x \;\; y \in [0, \pi/2) \cup [\pi, 3\pi/2)$$

$$y = \cot^{-1} x \quad x \in (-\infty, \infty) \quad \Leftrightarrow \cot y = x \quad y \in (0, \pi).$$

Warning: the choice of range is not universal; you might pick up a different book and find a different choice.

Derivatives of trigonometric functions

Memorize these:

$$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\csc^{-1}x = \frac{-1}{x\sqrt{x^2-1}}$$
$$\frac{d}{dx}\cos^{-1}x = \frac{-1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\sec^{-1}x = \frac{1}{x\sqrt{x^2-1}}$$
$$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$$
$$\frac{d}{dx}\cot^{-1}x = -\frac{1}{1+x^2}.$$