Section 4.4: L'Hôpital's rule

MATH 1700

L'Hôpital's rule for limits at a

Assume that (1) f and g are differentiable on an open interval I containing a (except possibly at a) and (2) $g'(x) \neq 0$ on I.
If **either**

$$\lim_{x\to a} f(x) = 0 \quad \text{and} \quad \lim_{x\to a} g(x) = 0$$

or

$$\lim_{x\to a} f(x) = \pm\infty \quad \text{and} \quad \lim_{x\to a} g(x) = \pm\infty$$

then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

so long as

$$\lim_{x\to a} \frac{f'(x)}{g'(x)} \quad \text{exists or is} \quad \pm\infty.$$

L'Hôpital's rule for right/left hand limits or limits at $\pm\infty$

In the previous slide, if you replace

$$\lim_{x\to a} f(x)$$

everywhere by

$$\lim_{x\to a^+} f(x), \quad \text{or} \quad \lim_{x\to a^-} f(x)$$

or

$$\lim_{x\to \pm\infty} f(x)$$

then the result is still true.

Indeterminate products

Inderminate products

If you need to find

$$\lim_{x \to a} f(x)g(x)$$

when

$$\lim_{x \to a} f(x) = 0 \quad \text{and} \quad \lim_{x \to a} g(x) = \pm\infty$$

try applying L'Hôpital's rule to

$$\frac{f(x)}{1/g(x)} \quad \text{or} \quad \frac{g(x)}{1/f(x)}.$$

Indeterminate products

Inderminate products
If you need to find

$$\lim_{x\to a} f(x)g(x)$$

when

$$\lim_{x\to a} f(x) = 0 \quad \text{and} \quad \lim_{x\to a} g(x) = \pm\infty$$

try applying L'Hôpital's rule to

$$\frac{f(x)}{1/g(x)} \quad \text{or} \quad \frac{g(x)}{1/f(x)}.$$

Same trick works for

$$\lim_{x\to\pm\infty} f(x)g(x)$$

if one goes to 0 and the other goes to $\pm\infty$.

Indeterminate differences

If you need to find

$$\lim\,[f(x) - g(x)] \quad \text{when} \quad f(x) \to \infty \;\text{ and }\; g(x) \to \infty,$$

play around with the expression algebraically until you can apply L'Hôpitals rule or possibly some other method.

Indeterminate powers

If you need to find

$$\lim f(x)^{g(x)}$$

when

$$\begin{aligned} &1. f \to 0 \ \text{ and } \ g \to 0 \\ \text{or } &2. f \to \infty \ \text{ and } \ g \to 0 \\ \text{or } &3. f \to 1 \ \text{ and } \ g \to \pm\infty \end{aligned}$$

try rewriting this as

$$f(x)^{g(x)} = e^{g(x) \ln f(x)}.$$