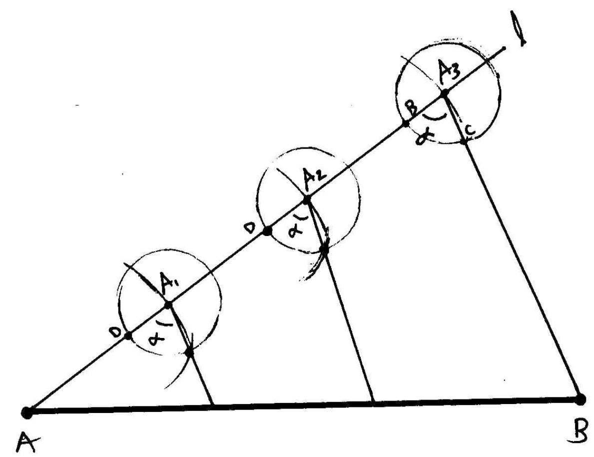
| 1. Subdivide the given line segment into 3 equal parts (using an unmarked ruler and a compass). Briefly describe your steps.



① Draw a line extending from A, call it l

② Cut l into 3 equal pieces.

③ Connect A_3 to B

④ Duplicat α at A_2, A_1

⑤ Extend the lines to cut AB, DONE!

Figure 1:

2] 2. (a) Prove that the triangle in Figure 2 is an acute golden triangle (HINT: Use the lengths of the sides)

a)

$$\frac{8.09}{5} = 1.618 = \varnothing$$

(the golden ratio)

b)

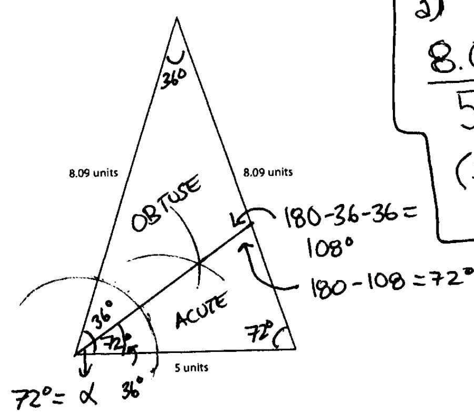


Figure 2:

[8] (b) Using an unmarked ruler and a compass, subdivide the given acute golden triangle into a pair of acute and obtuse golden triangles. Describe your steps and calculate the angles of the resulting triangles to justify your result.

① Bisect the angle α (this cuts 72° into 36° & 36°)

[7] 3. Given the 22nd and 25th Fibonacci numbers, $f_{22} = 17711$ and $f_{25} = 75025$, what are the 23rd and 24th Fibonacci numbers f_{23} and f_{24}? State the recursive formula for the Fibonacci numbers and use it to determine the two missing values (ie, DO NOT list the sequence all the way from f_1).

$$f_{22} + f_{23} = f_{24} \rightarrow 17711 + f_{23} = f_{24}$$

$$f_{23} + f_{24} = f_{25} \rightarrow f_{23} + f_{24} = 75025$$

$$\Rightarrow f_{23} + 17711 + f_{23} = 75025 \Rightarrow f_{23} = \frac{75025 - 17711}{2} =$$

$$\& \; f_{24} = 17711 + \qquad =$$

[5] 4. (a) One of the objects in Figure 3 is obtained from the other by rotation. Use an unmarked ruler and a compass to construct the center of the rotation.

① Connect 2 points & their images
② Bisect Af(A) & Bf(B)
③ Their intersection (of the bisectors) is the centre, O

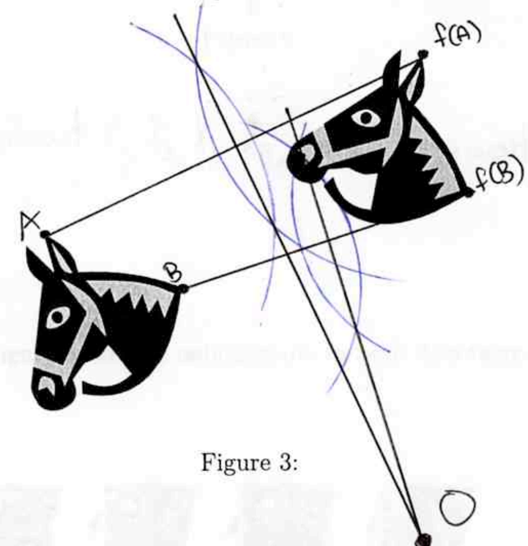
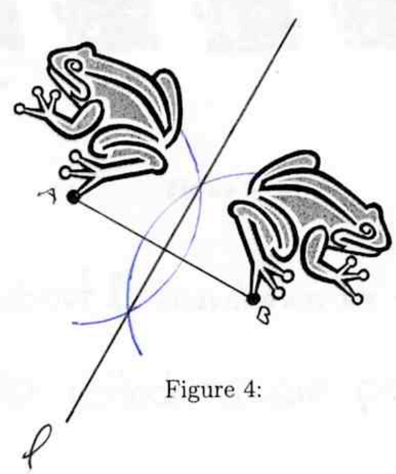


Figure 3:

[2] (b) One of the objects in Figure 4 is obtained from the other by reflection. Use an unmarked ruler and a compass to construct the line of reflection.



① Connect 2 corresponding points, A & B
② Bisect the line AB
③ This is the line of reflection

Figure 4:

5. Find the group of symmetries for each of the 2 objects shown below. If you claim there is a rotational symmetry, indicate the center of the rotation and the angle of rotation. If there are reflections, show the line(s) of reflection. If there are translational symmetries show or describe the vectors of translation.

[8] (a) For Figure 5 disregard the colouring (ie, assume the figure is all one colour)

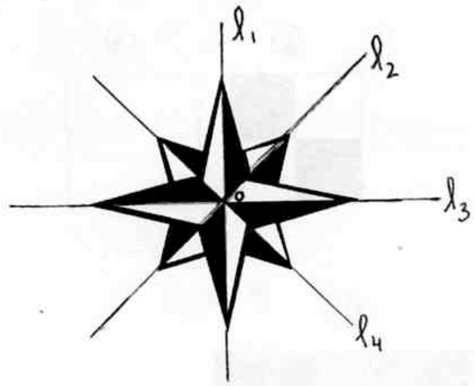


Figure 5:

$\{I$, reflection about $\ell_1, \ell_2, \ell_3, \ell_4$, rotation about O by $\frac{360}{4} = 90°$, $180°, 270°\}$

[4] (b) Assume Figure 6 extends unboundedly in both directions.

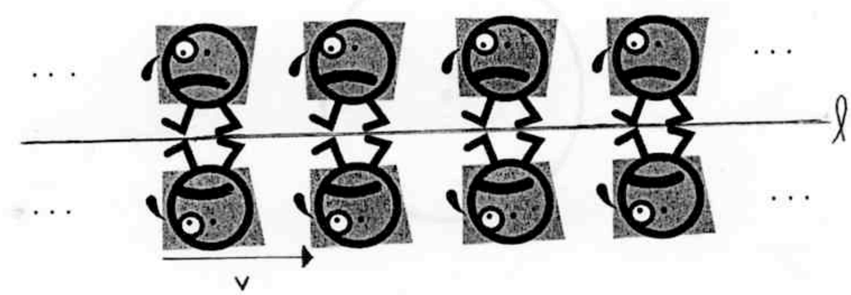


Figure 6:

$\{I$, reflection about ℓ, translation by $v, 2v, 3v, \cdots$ & $-v, -2v, -3v \cdots\}$
(also, the glide reflections are present)

[6] 6. The first two stages of a construction of a fractal are given in the picture below. The shape on the left has been cut into 4 equal parts, and then the top left and bottom right pieces have been eliminated in order to get the shape on the right. Draw the shapes obtained after applying this procedure once, and then once more.

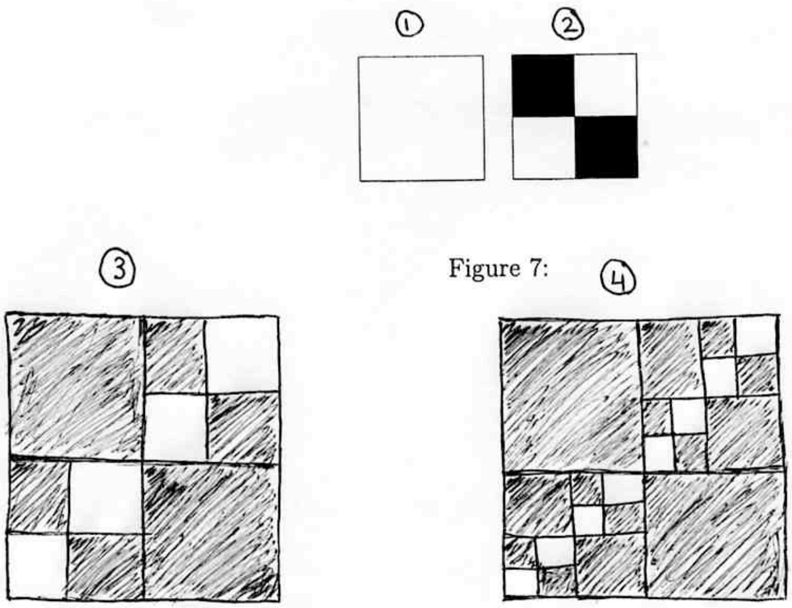


Figure 7:

7. ***BONUS*** [4] How many symmetries does the circle (below) have? Identify them.

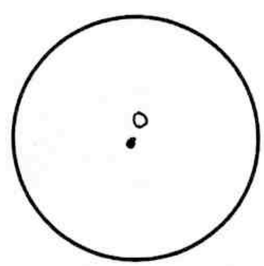


Figure 8:

{I, rotation about O by <u>any</u> angle from 0° to 360°, reflection about <u>any</u> straight line through O}

So there are an infinite number of symmetries!