

THE UNIVERSITY OF MANITOBA

February 23, 2007

MIDTERM EXAMINATION
TITLE PAGE

DEPARTMENT & COURSE NO: MATH 1520
EXAMINATION: Intro Calc. for Mgmt & Soc Sci

TIME: 1 HOUR
EXAMINER: (Identified Below)

LAST NAME: (PRINT) _____

FIRST NAME: (PRINT) _____

STUDENT NUMBER: _____

SIGNATURE: _____

(I understand that cheating is a serious offense)

Please indicate your instructor and section by placing a check mark in the appropriate box below.

<u>SECTION</u>		<u>TIME</u>	<u>INSTRUCTOR</u>
<input type="checkbox"/> A01	M, W, F Thurs	2:30- 3:20 1:00- 1:50	W. Korytowski
<input type="checkbox"/> A02	M, W, F	11:30- 12:20	D. Kalajdziewska
<input type="checkbox"/> A03	Tues, Thurs	11:30- 12:45	W. Korytowski

INSTRUCTIONS TO STUDENTS:

Calculators and other aids such as textbooks, notes, cell phones, translators, etc, are not allowed.

This is a 1 hour exam. Please show your work clearly. Please justify your answers, unless otherwise stated.

This exam has a title page, 4 pages of questions and 1 blank sheet at the end of the paper for rough work if necessary. Please check that you have all the pages. The blank page may be removed if desired but please do so carefully so you don't remove the staple.

The value of each question is indicated in the left-hand margin beside the statement of the question. The total value of all questions is 60.

Answer all questions in the space provided. If necessary, you may continue your work on the reverse sides of the pages, but please indicate clearly your work is continued elsewhere.

DO NOT WRITE IN THIS COLUMN

1. _____ /10

2. _____ /4

3. _____ /5

4. _____ /10

5. _____ /6

6. _____ /6

7. _____ /15

8. _____ /4

TOTAL

_____ /60

DATE: February 23, 2005

TIME: 1 Hour

DEPARTMENT & COURSE NO: 136.1520

EXAMINERS: Staff

EXAMINATION: Calculus

Answer all questions.

Clearly show all work and justify your answers.

Value

1. A company produces hockey pucks with its monthly revenue and cost (in thousands of dollars) are

$$R(x) = 2x^2 + x + 10$$

$$C(x) = 3x + 50$$

- (a) Find an expression for the profit function.

$$P(x) = R(x) - C(x) = 2x^2 + x + 10 - 3x - 50 = 2x^2 - 2x - 40$$

- (b) What is the company's fixed cost?

$$\$50$$

- (c) Determine the company's break-even number of pucks.

$$R(x) = C(x), \quad 2x^2 + x + 10 = 3x + 50$$

$$\Rightarrow 2x^2 - 2x - 40 = 0 \Rightarrow (2x+8)(x-5) = 0$$

$$\Rightarrow x = -8/2 = -4 \quad (\text{NO! -ve \# doesn't make sense})$$

- (d) Find the marginal profit if 1000 hockey pucks are sold.

$$\Rightarrow x = 5$$

$$\text{Marginal Profit} = P'(x)$$

$$= 4x - 2 = 2(2x - 1)$$

$$P'(1000) = 4(1000) - 2 = 4000 - 2 = \$3998.00$$

2. Mani Toba invested \$500 in an account earning 4% compounded monthly. Assuming he does not make any further deposits or withdrawals from this account, how long will it take for this investment to double itself? [LEAVE YOUR ANSWER IN LOGARITHMIC FORM]

$$1000 = 500 \left(1 + \frac{0.04}{12}\right)^{12t}$$

$$2 = \left(1 + \frac{0.04}{12}\right)^{12t} \Rightarrow \ln 2 = \ln \left(1 + \frac{0.04}{12}\right)^{12t}$$

$$\Rightarrow \ln 2 = 12t \cdot \ln \left(1 + \frac{0.04}{12}\right)$$

$$\Rightarrow t = \frac{\ln 2}{12 \ln \left(1 + \frac{0.04}{12}\right)}$$

Value

3. The kangaroo population in Australia is said to be growing exponentially. This population was estimated to be 6 million in 1996 and 24 million in 2006. If this growth rate continues, what is the projected kangaroo population for 2011?

Call 1996 the initial year, year 0

5 then 2006 is year 10

$$\Rightarrow \text{using } y = y_0 e^{kt}, \quad 24 = 6e^{10k}$$

$$\Rightarrow 4 = e^{10k} \quad \& \quad \ln 4 = \ln e^{10k} = 10k$$

$$\Rightarrow k = \frac{\ln 4}{10} \quad \& \quad y = y_0 e^{\frac{\ln 4}{10} t}$$

year 15
↓

in 2011

$$\Rightarrow y = 6e^{\frac{\ln 4}{10} \cdot 15} = 6e^{\frac{3}{2} \ln 4} = 6e^{\ln(4^{3/2})} = 6e^{\ln 8} = 6 \cdot 8$$

= 48 million.

4. Find the following limits, if they exist.

$$(a) \lim_{x \rightarrow 3} \frac{x^3 - 9x}{x^2 - 4x + 3} = \lim_{x \rightarrow 3} \frac{x(x-3)(x+3)}{(x-3)(x-1)}$$

$$3 \quad = \frac{3(6)}{2} = 9$$

$$(b) \lim_{x \rightarrow 8} \frac{3 - \sqrt{x+1}}{x^2 - 8x} \cdot \frac{3 + \sqrt{x+1}}{3 + \sqrt{x+1}} = \lim_{x \rightarrow 8} \frac{9 - (x+1)}{x(x-8)(3 + \sqrt{x+1})}$$

$$4 \quad = \lim_{x \rightarrow 8} \frac{8-x}{x(x-8)(3 + \sqrt{x+1})} = \lim_{x \rightarrow 8} \frac{-(x-8)}{x(x-8)(3 + \sqrt{x+1})}$$

$$= \frac{-1}{8(3 + \sqrt{9})} = \frac{-1}{48}$$

$$(c) \lim_{x \rightarrow \infty} \frac{(3x+2)(2x-5)}{x^2+1000} = \lim_{x \rightarrow \infty} \frac{6x^2 - 11x - 10}{x^2 + 1000}$$

$$3 \quad = \lim_{x \rightarrow \infty} \frac{6x^2/x^2 - 11x/x - 10/x^2}{x^2/x^2 + 1000/x^2} = 6$$

Value

5. Given the function defined by $f(x) = \begin{cases} x^2 - 1 & x < 3 \\ 9 & x = 3 \\ 2x + 2 & x > 3 \end{cases}$

(a) Determine if $\lim_{x \rightarrow 3} f(x)$ exists. Justify your answer.

$$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3} x^2 - 1 = 9 - 1 = 8$$

$$\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3} 2x + 2 = 6 + 2 = 8$$

So $\lim_{x \rightarrow 3} f(x) = 8$ exists.

(b) Is $f(x)$ continuous, or discontinuous, at $x = 3$? Justify.

$f(3) = 9 \neq \lim_{x \rightarrow 3} f(x) = 8$ so $f(x)$ is discontinuous at $x = 3$.

6. Use the definition of the derivative to find $f'(x)$ if $f(x) = \frac{2}{3-x}$.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{2}{3-(x+h)} - \frac{2}{3-x}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2(3-x) - 2(3-x-h)}{(3-x)(3-x-h)} \cdot \frac{1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{6 - 2x - 6 + 2x + 2h}{h(3-x)(3-x-h)}$$

$$= \lim_{h \rightarrow 0} \frac{2}{(3-x)(3-x-h)} = \frac{2}{(3-x)^2}$$

Value

7. For each function below, find y' . [DO NOT SIMPLIFY YOUR ANSWERS.]

(a) $y = 4x^5 - \sqrt[3]{x} + x^{\frac{2}{3}} + \frac{1}{\sqrt[4]{x}}$

(b) $y = \frac{\sqrt{x^4 + x}}{(5x^2 - 6x)^2}$

(c) $y = (x^3 - 6x + 5)^{10} \left(x - \frac{1}{x}\right)^5$

extra
practice

8. Find an equation of the line tangent to the curve $y = x^3 - 2x^2 + 5$ at the point on the curve where $x = 2$.

$$y' = 3x^2 - 4x, \quad y'(2) = 3(4) - 8 = 4$$

↖
slope of tangent line

if $x = 2$, $y = 8 - 8 + 5 = 5$

⇒ eqn' of tangent line is

$$y - 5 = 4(x - 2)$$