

UNIVERSITY OF MANITOBA

DATE: December 14, 2006

FINAL EXAMINATION

PAPER # 406

PAGE: 1 of 7

DEPARTMENT & COURSE NO: MATH 1500

TIME: 2 hours

EXAMINATION: Intro Calculus

EXAMINER: Various

[25] 1. Find $\frac{dy}{dx}$ for the following (DO NOT SIMPLIFY):

(a) $y = e^{5x} \ln(2 - x^3)$

(b) $y = \frac{x^2}{2x}$

(c) $y = (\ln x)^{x^2}$

Question	Points	Score
1	25	
2	10	
3	30	
4	7	
5	7	
6	14	
7	4	
8	12	
9	22	
Total	139	

(d) $x^y = y^x$

INSTRUCTIONS TO STUDENTS:

This is a 2 hour exam. Please show your work clearly.

No text, notes or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a time limit of 2 hours. Questions and answers are on separate pages. Please check that you have all the pages. You may remove the blank page if you wish, but do not remove the marks.

The value of each question is indicated in the table below. The total value of all questions is 139 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

UNIVERSITY OF MANITOBA

DATE: December 14, 2006

FINAL EXAMINATION

PAPER # 406

PAGE: 2 of 7

DEPARTMENT & COURSE NO: MATH 1500

TIME: 2 hours

EXAMINATION: Intro Calculus

EXAMINER: Various

[10] 2. Prove the following: If $f'(x) = 0$ for all x in an interval I , then f is constant on I .

UNIVERSITY OF MANITOBA

DATE: December 14, 2006

FINAL EXAMINATION

PAPER # 406

PAGE: 3 of 7

DEPARTMENT & COURSE NO: MATH 1500

TIME: 2 hours

EXAMINATION: Intro Calculus

EXAMINER: Various

[30] 3. If $f(x) = \frac{8(x-2)}{x^2}$ then $f'(x) = \frac{-8(x-4)}{x^3}$ and $f''(x) = \frac{16(x-6)}{x^4}$.

(a) Find all intercepts of the function; state the domain.

(b) Calculate all limits associated with any horizontal and vertical asymptotes to the curve $y = f(x)$. Also, give the equations of these asymptotes, if any.

(c) Find the critical points of $f(x)$, the intervals where $f(x)$ is increasing and the intervals where $f(x)$ is decreasing. Find the coordinates of any local maxima and/or minima of $f(x)$.

UNIVERSITY OF MANITOBA

DATE: December 14, 2006

FINAL EXAMINATION

PAPER # 406

PAGE: 4 of 7

DEPARTMENT & COURSE NO: MATH 1500

TIME: 2 hours

EXAMINATION: Intro Calculus

EXAMINER: Various

(d) Find where the $f(x)$ is concave up, where $f(x)$ is concave down. Find inflection points if any.

(e) Sketch the curve $y = f(x)$, displaying the information found in parts (a),(b), (c) and, (d).

UNIVERSITY OF MANITOBA

DATE: December 14, 2006

FINAL EXAMINATION

PAPER # 406

PAGE: 5 of 7

DEPARTMENT & COURSE NO: MATH 1500TIME: 2 hoursEXAMINATION: Intro CalculusEXAMINER: Various

For the following two questions it is sufficient to write down a function and indicate an interval on which it is defined. The function and interval should be chosen so that in order to solve the problem you would find the maximum or minimum value of the function on that interval. **IT IS NOT NECESSARY TO ACTUALLY FIND THE MAXIMUM OR MINIMUM VALUE.**

- [7] 4. Two equal adjacent rectangular areas are to be created with fencing. (The two areas are to be separated with one line of fencing.) If 100 m of fence is available, what is the largest total area which can be fenced?

- [7] 5. A piece of wire 10 cm long is to be cut into two pieces. One piece is bent into an equilateral triangle and the other into a square. How should the wire be cut to maximize the area? You are allowed to use all the wire for one shape and none of the wire for another (that is to not cut the wire). (The area of an equilateral triangle is $\frac{\sqrt{3}}{4}s^2$ where s is the length of a side.)

UNIVERSITY OF MANITOBA

DATE: December 14, 2006

FINAL EXAMINATION

PAPER # 406

PAGE: 6 of 7

DEPARTMENT & COURSE NO: MATH 1500TIME: 2 hoursEXAMINATION: Intro CalculusEXAMINER: Various

- [13] 6. Find the absolute maximum and absolute minimum values of the function $f(x) = 2x^3 + 3x^2 - 12x + 1$ on the interval $[-1, 2]$.

- [4] 7. Let $f(x) = \int_0^x (e^{\cos t} - \tan t) dt$ for $-\pi/2 < x < \pi/2$. Find $f'(0)$.

- [12] 8. Find the total area enclosed by the curve $y = x(x-1)(x+1)$ and the x -axis.

UNIVERSITY OF MANITOBA

DATE: December 14, 2006

FINAL EXAMINATION

PAPER # 406

PAGE: 7 of 7

DEPARTMENT & COURSE NO: MATH 1500TIME: 2 hoursEXAMINATION: Intro CalculusEXAMINER: Various

[12] 9. Evaluate the following integrals:

(a)
$$\int (x^4 - \frac{1}{x^2} + e^x + \sec^2(2x)) dx$$

(b)
$$\int_{-1}^1 (x^3 - x) dx$$