Formulas/Strategies

Logarithmic Functions:

1. $f(x) = \log_a x \quad for\ x > 0$
2. $y = \log_a x \Leftrightarrow a^y = x$
3. $\log_{10} x = \log x$
4. $\log_e x = \ln x$
   - i) $\ln e = 1$
   - ii) $\ln 1 = 0$
5. $\log_a(xy) = \log_a x + \log_a y$
6. $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$
7. $\log_a(x^r) = r\log_a x \quad [NOT \quad (\log_a x)^r = r\log_a x]$
8. $\log_a a = 1 \Rightarrow \log_a(a^r) = r\log_a a = r$
9. $\log_a 1 = 0$
10. Change of Base (Logs): $\log_a x = \frac{\log_b x}{\log_b a} = \frac{\ln x}{\ln a}$

Derivatives of Logs/Exponentials:

$$[a^{g(x)}]' = (a^{g(x)})(g'(x))(\ln a) \Rightarrow [a^x]' = (a^x)(\ln a)$$

$$[e^{g(x)}]' = (e^{g(x)})(g'(x)) \Rightarrow [e^x]' = e^x$$

$$[\log_a|g(x)|]' = \frac{g'(x)}{g(x)\ln a} \Rightarrow [\ln|g(x)|]' = \frac{g'(x)}{g(x)} \Rightarrow [\ln|x|]' = \frac{1}{x}$$

Curve Sketching:

1. Consider the domain of f(x) and note any restrictions
2. x - intercept at $y = 0$, y - intercept at $x = 0$
3. Find asymptotes:
   - a) vertical if denominator $= 0$
   - b) horizontal if $\lim_{x \to \pm\infty} f(x)$ exists
4. Find critical points $x = c$ where $f'(x) = 0$ or $f'(x)$ d.n.e.
   - a) increasing where $f'(x) > 0$
   - b) decreasing where $f'(x) < 0$
5. Find relative extrema using part 4 or
   - a) $f''(c) > 0 \Rightarrow$ f(c) is a relative min at $x = c$
   - b) $f''(c) < 0 \Rightarrow$ f(c) is a relative max at $x = c$
6. Find inflection points where $f''(x) = 0$ or $f''(x)$ d.n.e.
   - a) concave up where $f''(x) > 0$
   - b) concave down where $f''(x) < 0$
7. Plot and connect all important points

Max/Min Problems:

1. Read the problem carefully, sketch if you can
2. Decide which variable (equation) to maximize or minimize, f(x)
3. Write this equation in terms of ONE variable
4. State the domain of f(x) in terms of this variable
5. Find f'(x), and the critical points and endpoints of f(x)
6. Test them all by plugging into f(x)
7. The absolute max is the largest of these values,
   the absolute min is the smallest of these values
8. Write your answer in the form of a SENTENCE!

Area Formulas:

Area = (length)(width)

Volume = (length)(width)(height)

Area of a Triangle = $1/2$ (base)(height)

Area of a Circle = πr^2 (where r is the radius)

Circumference of a Circle = $2\pi r$

Antiderivatives:

1. $\int kf(x)dx = k\int f(x)dx$ (k any constant)

2. $\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$

3. $\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad if\ n \neq -1$

4. $\int x^{-1}dx = \int \frac{1}{x}dx = \ln|x| + C$

5. $\int e^{ax}dx = \frac{e^{ax}}{a} + C$

6. $\int \sin(ax)dx = \frac{-\cos(ax)}{a} + C$

7. $\int \cos(ax)dx = \frac{\sin(ax)}{a} + C$

8. $\int \sec^2 x dx = \tan x + C$

9. $\int_a^b f(x)dx = F(b) - F(a)$ where F(x) is the antiderivative of f(x)

10. If the definate integral above represents area, it must be positive, so find regions where f(x) < 0 and take $-\int_a^b f(x)dx$ for those regions (if you are using area to solve an integral, areas below the x - axis will give a negative - valued integral).