Curve Sketching Problems

For the following functions, f, find all local extrema, inflection points, vertical and horizontal asymptotes. Sketch the graph of f. Find the domain and range of f.

1. a) $f(x) = x^3 - 9x^2 + 15x$ $[f'(x) = 3(x-1)(x-5),\ f''(x) = 6(x-3)]$

   b) $f(x) = 5 - 9x + 6x^2 - x^3$ $[f'(x) = -3(x-1)(x-3),\ f''(x) = -6(x-2)]$

   c) $f(x) = x^3 - 3x^2 + 3x$ $[f'(x) = 3(x-1)^2,\ f''(x) = 6(x-1)]$

   d) $f(x) = x^4 - 6x^2$ $[f'(x) = 4x(x^2-3),\ f''(x) = 12(x^2-1)]$

   e) $f(x) = -x^4 - 4x^3 + 16x$ $[f'(x) = -4(x-1)(x+2)^2,\ f''(x) = -12x(x+2)]$

   f) $f(x) = -3x^5 + 10x^3 - 15x$ $[f'(x) = -15(x-1)^2(x+1)^2,\ f''(x) = -60x(x^2-1)]$

   g) $f(x) = (x^2-1)^3$ $\left[f'(x) = 6x(x-1)^2(x+1)^2,\ f''(x) = 30(x^2-1)(x^2-\frac{1}{5})\right]$

2. a) $f(x) = \dfrac{3}{3+x^2}$ $\left[f'(x) = \dfrac{-6x}{(3+x^2)^2},\ f''(x) = \dfrac{18(x+1)(x-1)}{(3+x^2)^3}\right]$

   b) $f(x) = \dfrac{x}{x^2+1}$ $\left[f'(x) = \dfrac{1-x^2}{(x^2+1)^2},\ f''(x) = \dfrac{2x(x^2-3)}{(x^2+1)^3}\right]$

   c) $f(x) = \dfrac{x^2}{x+1}$ $\left[f'(x) = \dfrac{x^2+2x}{(x+1)^2},\ f''(x) = \dfrac{2}{(x+1)^3}\right]$

   d) $f(x) = \dfrac{4(x+1)}{x^2}$ $\left[f'(x) = \dfrac{-4(x+2)}{x^3},\ f''(x) = \dfrac{8(x+3)}{x^4}\right]$

   e) $f(x) = \dfrac{-3(x-1)^2}{(x+1)^2}$ $\left[f'(x) = \dfrac{-12(x-1)}{(x+1)^3},\ f''(x) = \dfrac{24(x-2)}{(x+1)^4}\right]$

   f) $f(x) = \dfrac{x^3}{x^2-4}$ $\left[f'(x) = \dfrac{x^2(x^2-12)}{(x-2)^2(x+2)^2},\ f''(x) = \dfrac{8x(x^2+12)}{(x-2)^3(x+2)^3}\right]$

   g) $f(x) = \dfrac{3x^2}{x^2-9}$ $\left[f'(x) = \dfrac{-54x}{(x^2-9)^2},\ f''(x) = \dfrac{162(x^2+3)}{(x^2-9)^3}\right]$

   h) $f(x) = \dfrac{9(x^2-3)}{x^3}$ $\left[f'(x) = \dfrac{-9(x^2-9)}{x^4},\ f''(x) = \dfrac{18(x^2-18)}{x^5}\right]$

3. a) $f(x) = \dfrac{1}{\sqrt{x^2+1}}$ $\left[f'(x) = \dfrac{-x}{(x^2+1)^{3/2}},\ f''(x) = \dfrac{2x^2-1}{(x^2+1)^{5/2}}\right]$

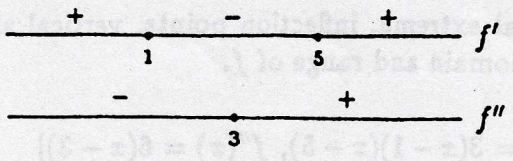
   b) $f(x) = \dfrac{x^2+3}{\sqrt{x^2+1}}$ $\left[f'(x) = \dfrac{x(x^2-1)}{(x^2+1)^{3/2}},\ f''(x) = \dfrac{5x^2-1}{(x^2+1)^{5/2}}\right]$

   c) $f(x) = x^{1/3}(x+4)$ $\left[f'(x) = \dfrac{4(x+1)}{3x^{2/3}},\ f''(x) = \dfrac{4(x-2)}{9x^{5/3}}\right]$

   d) $f(x) = x^{2/3}(5-x)$ $\left[f'(x) = \dfrac{5(2-x)}{3x^{1/3}},\ f''(x) = \dfrac{-10(x+1)}{9x^{4/3}}\right]$

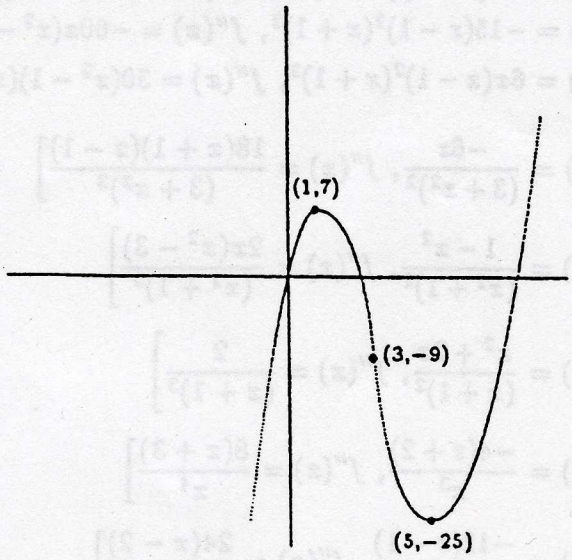
   e) $f(x) = \sqrt{\dfrac{4-x}{4+x}}$ $\left[f'(x) = \dfrac{-4}{\sqrt{(4-x)(4+x)^3}},\ f''(x) = \dfrac{8(2-x)}{\sqrt{(4-x)^3(4+x)^5}}\right]$

1.a)

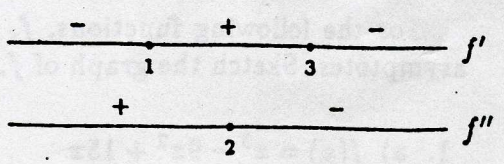


loc max: (1, 7)
loc min: (5, −25)
infl pts: (3, −9)

dom: $(-\infty, \infty)$
range: $(-\infty, \infty)$

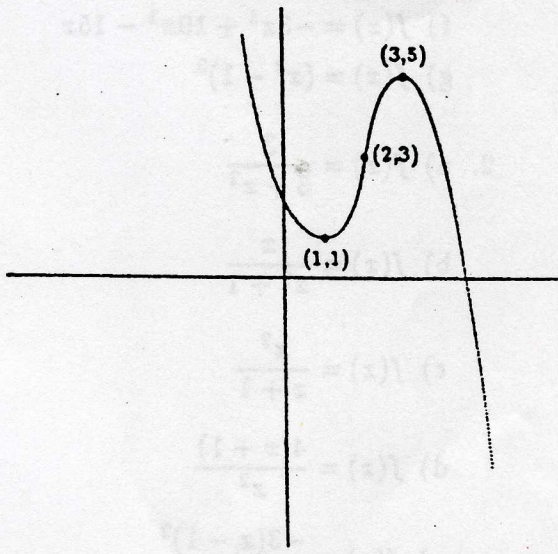


1.b)

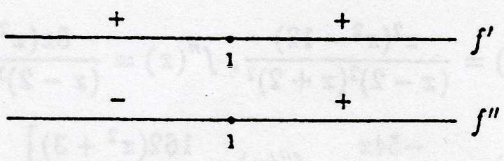


loc max: (3, 5)
loc min: (1, 1)
infl pts: (2, 3)

dom: $(-\infty, \infty)$
range: $(-\infty, \infty)$

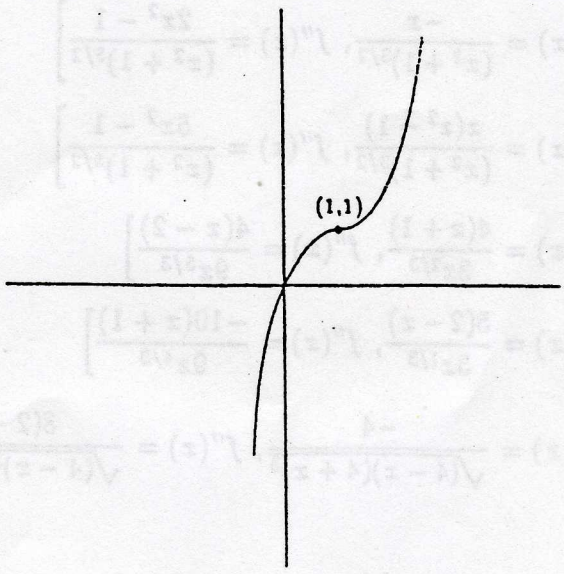


1.c)

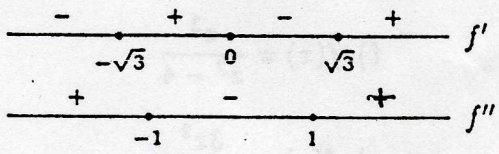


loc max: none
loc min: none
infl pts: (1, 1)

dom: $(-\infty, \infty)$
range: $(-\infty, \infty)$

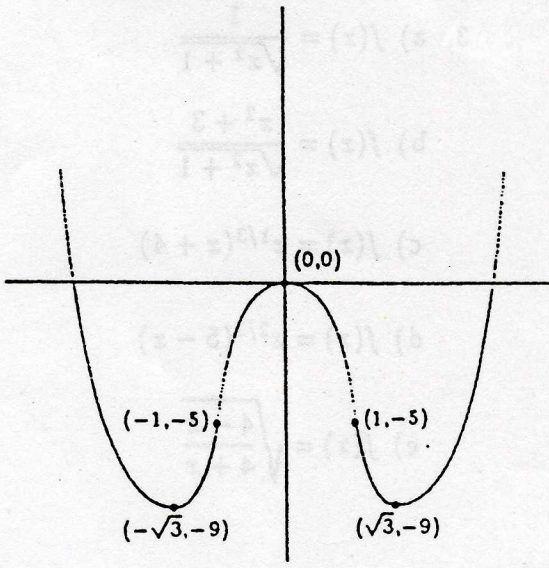


1.d)

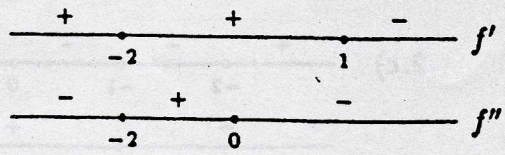


loc max: (0, 0)
loc min: $(-\sqrt{3}, -9), (\sqrt{3}, -9)$
infl pts: $(-1, -5), (1, -5)$

dom: $(-\infty, \infty)$
range: $[-9, \infty)$

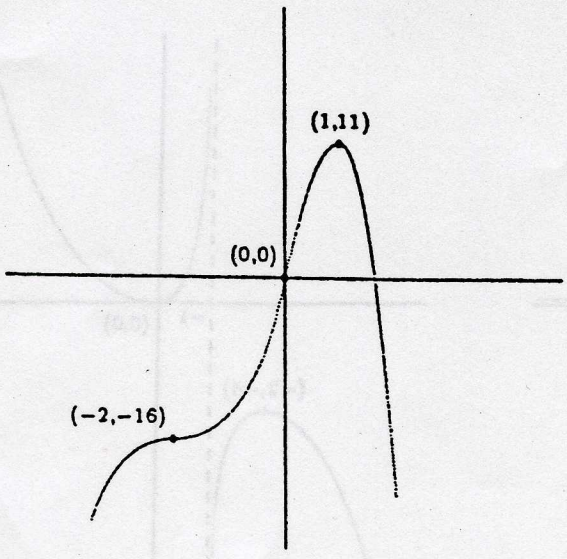


1.e)

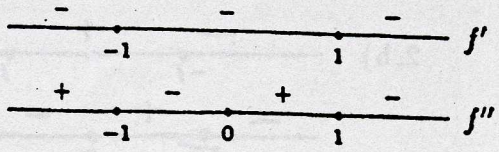


loc max: (1,11)
loc min: none
infl pts: (0,0),(−2,−16)

dom: $(-\infty,\infty)$
range: $(-\infty,11]$

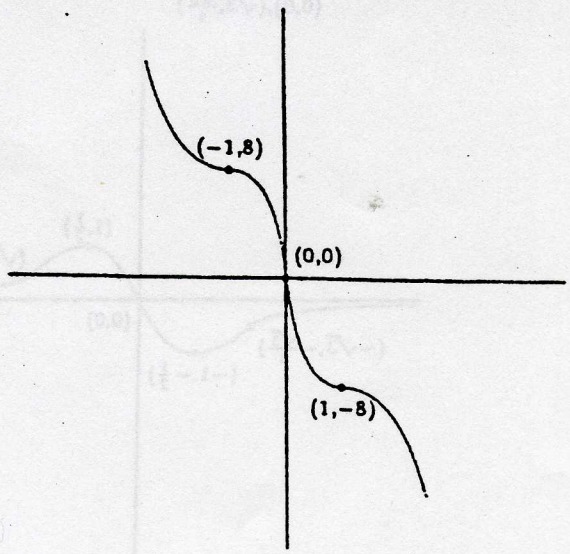


1.f)

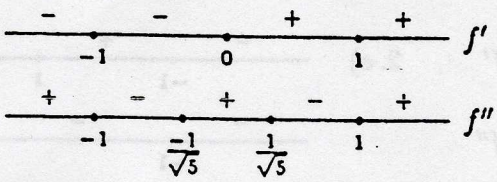


loc max: none
loc min: none
infl pts: (−1,8),(1,−8)

dom: $(-\infty,\infty)$
range: $(-\infty,\infty)$

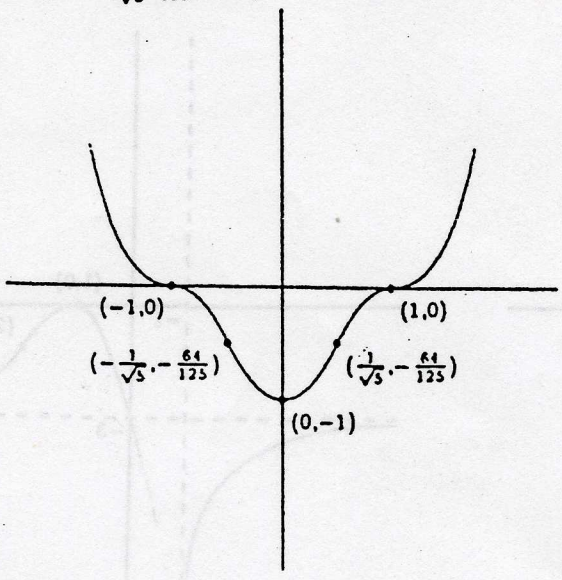


1.g)

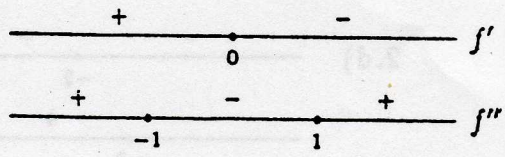


loc max: none
loc min: (0,−1)
infl pts: $(-1,0),(\frac{-1}{\sqrt{5}},\frac{-64}{125}),(\frac{1}{\sqrt{5}},\frac{-64}{125}),(1,0)$

dom: $(-\infty,\infty)$
range: $[-1,\infty)$

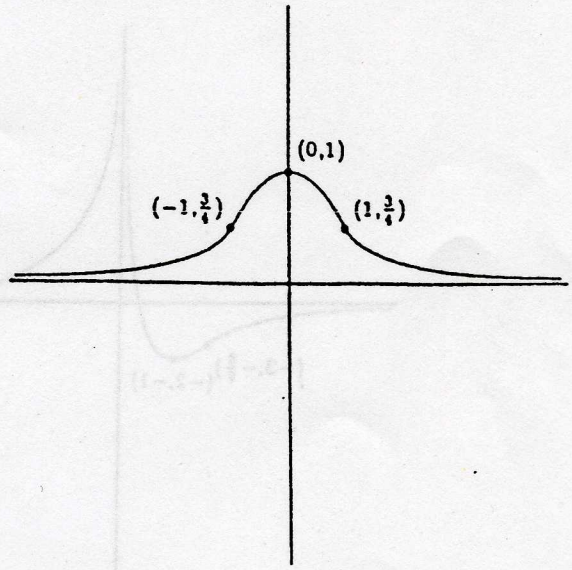


2.a)

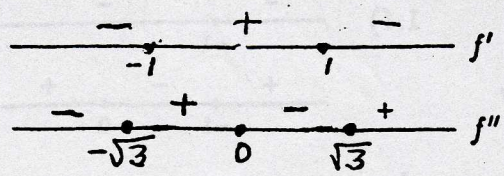


loc max: (0,1)
loc min: none
infl pts: $(-1,\frac{3}{4}),(1,\frac{3}{4})$

dom: $(-\infty,\infty)$
range: $(0,1]$

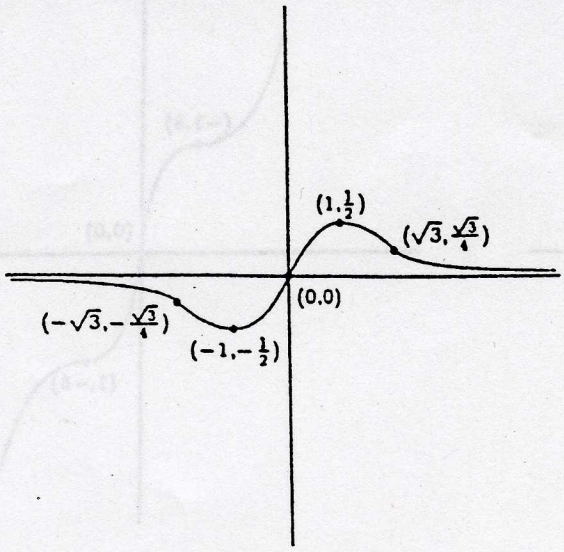


2. b)

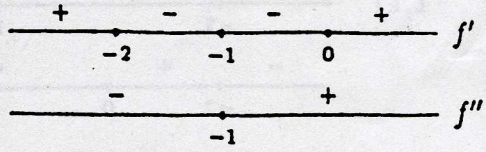


loc max: $(1, \frac{1}{2})$
loc min: $(-1, -\frac{1}{2})$
infl pts: $(-\sqrt{3}, -\frac{\sqrt{3}}{4})$, $(0,0)$, $(\sqrt{3}, \frac{\sqrt{3}}{4})$

dom: $(-\infty, \infty)$
range: $[-\frac{1}{2}, \frac{1}{2}]$

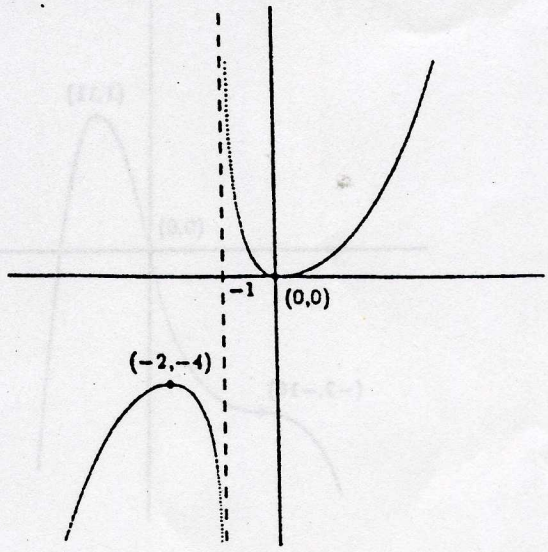


2. c)

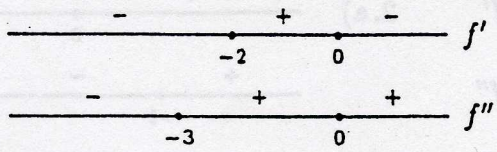


loc max: $(-2, -4)$
loc min: $(0,0)$
infl pts: none

dom: $(-\infty,-1) \cup (-1,\infty)$
range: $(-\infty,-4] \cup [0,\infty)$

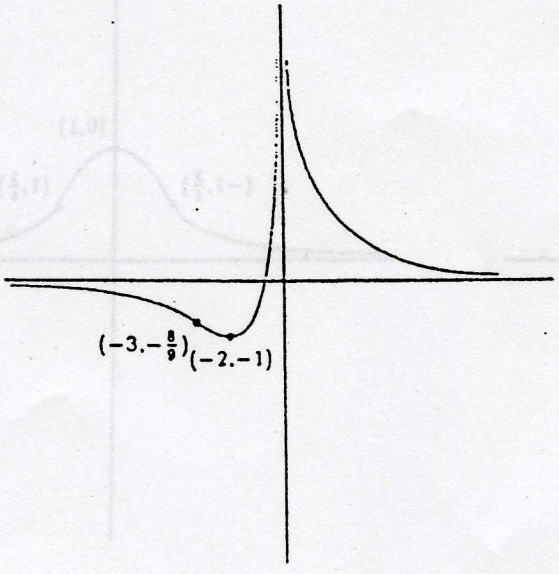


2. d)

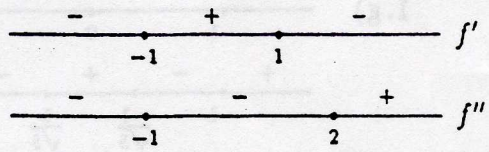


loc max: none
loc min: $(-2, -1)$
infl pts: $(3, -\frac{8}{9})$

dom: $\{x \neq 0\}$
range: $[-1, \infty)$

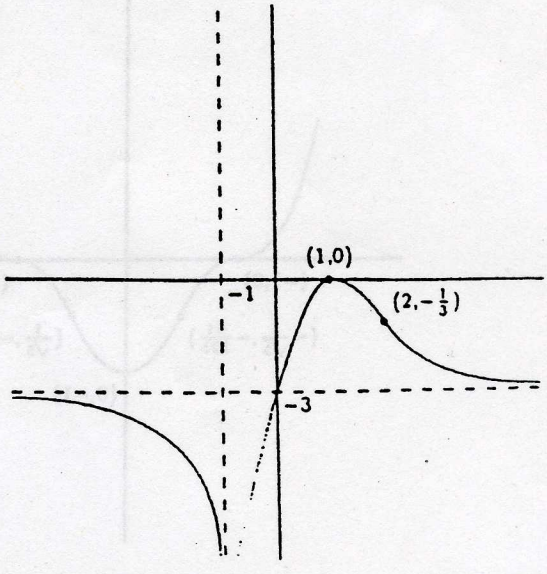


2. e)

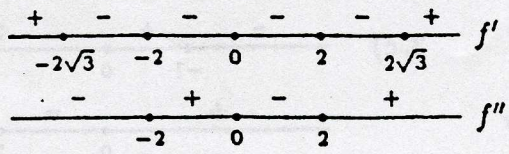


loc max: $(1,0)$
loc min: none
infl pts: $(2, -\frac{1}{3})$

dom: $\{x \neq -1\}$
range: $(\infty, 0]$

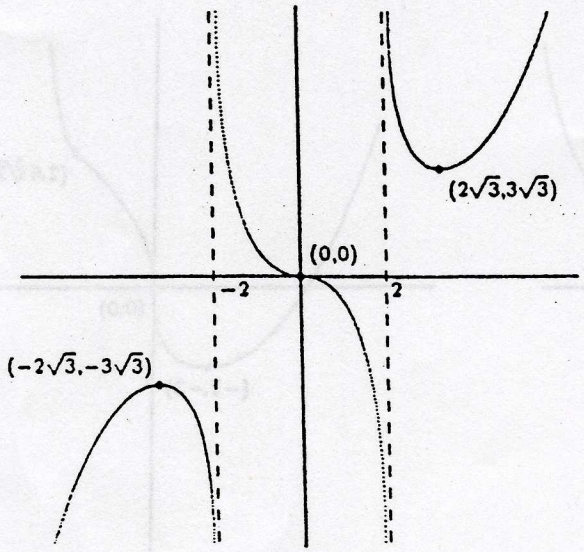


2.f)

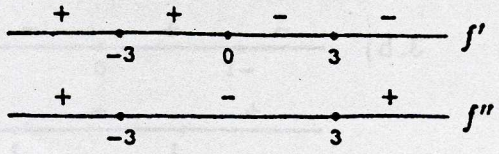


loc max: $(-2\sqrt{3},-3\sqrt{3})$
loc min: $(2\sqrt{3},3\sqrt{3})$
infl pts: $(0,0)$

dom: $\{x \neq -2,2\}$
range: $(-\infty,\infty)$

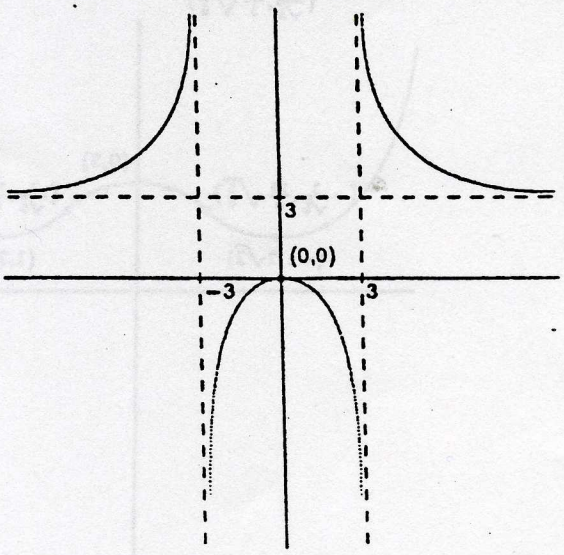


2.g)

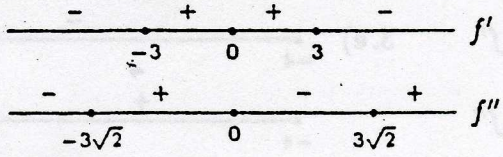


loc max: $(0,0)$
loc min: none
infl pts: none

dom: $\{x \neq -3,3\}$
range: $(-\infty,0]\cup(3,\infty)$

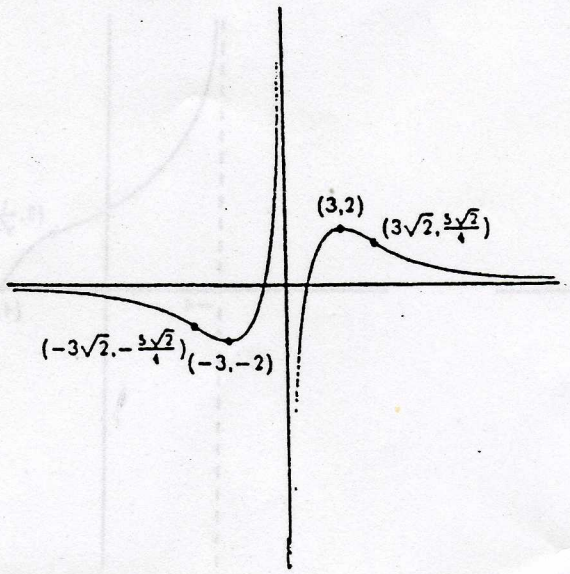


2.h)

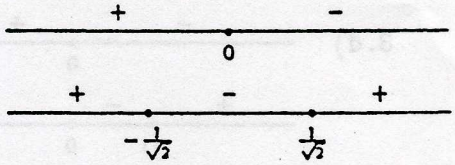


loc max: $(3,2)$
loc min: $(-3,-2)$
infl pts: $(-3\sqrt{2},-\frac{3\sqrt{2}}{4})$, $(3\sqrt{2},\frac{3\sqrt{2}}{4})$

dom: $\{x \neq 0\}$
range: $(-\infty,\infty)$

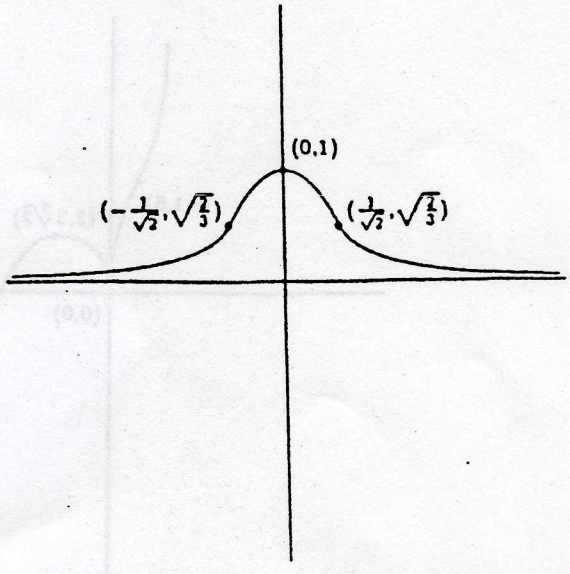


3.a)

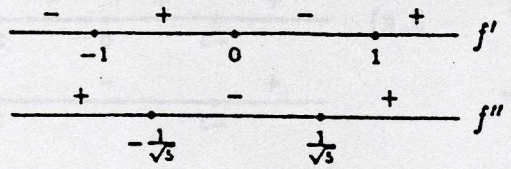


loc max: $(0,1)$
loc min: none
infl pts: $(-\frac{1}{2},-\sqrt{\frac{2}{3}}),(\frac{1}{2},\sqrt{\frac{2}{3}})$

dom: $(-\infty,\infty)$
range: $(0,1]$

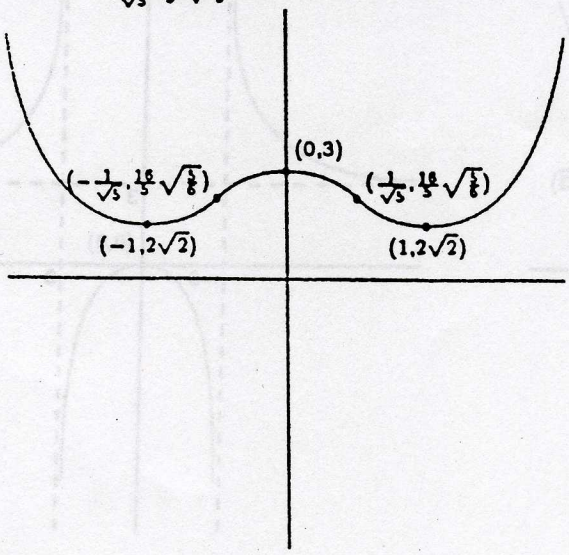


3. b)

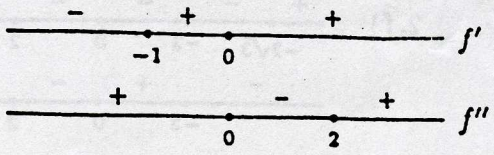


loc max: $(0,3)$
loc min: $(-1,2\sqrt{2}),(1,2\sqrt{2})$
infl pts: $(-\frac{1}{\sqrt{5}},\frac{16}{5}\sqrt{\frac{5}{6}})$, $(\frac{1}{\sqrt{5}},\frac{16}{5}\sqrt{\frac{5}{6}})$

dom: $(-\infty,\infty)$
range: $[2\sqrt{2},\infty)$

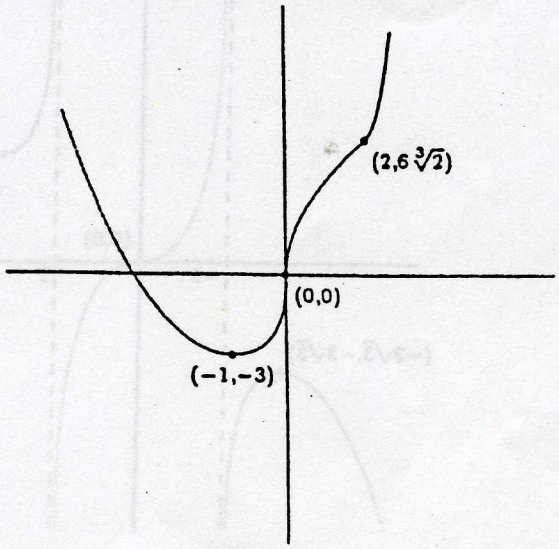


3. c)

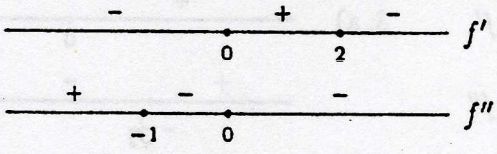


loc max: none
loc min: $(-1,-3)$
infl pts: $(0,0),(2,6\sqrt[3]{2})$

dom: $(-\infty,\infty)$
range: $[-3,\infty)$

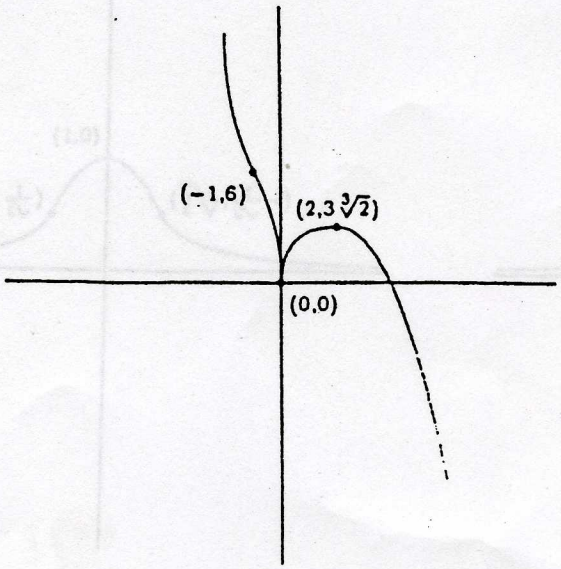


3. d)

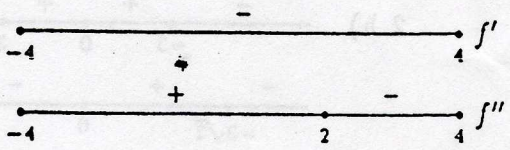


loc max: $(2,3\sqrt[3]{2})$
loc min: $(0,0)$
infl pts: $(-1,6)$

dom: $(-\infty,\infty)$
range: $(-\infty,\infty)$



3. e)



loc max: none
loc min: none
infl pts: $(2,\frac{1}{\sqrt{3}})$

dom: $(-4,4]$
range: $[0,\infty)$