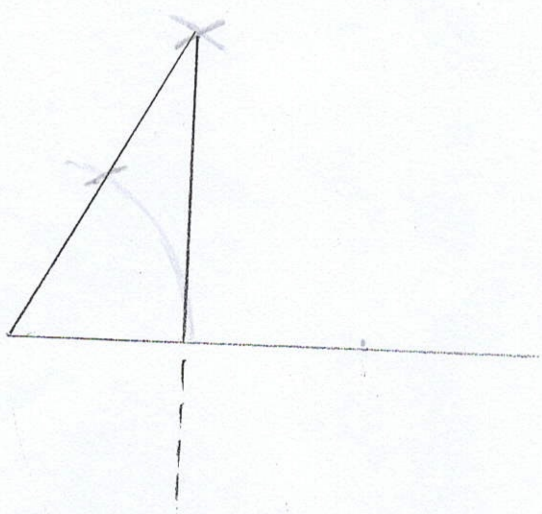
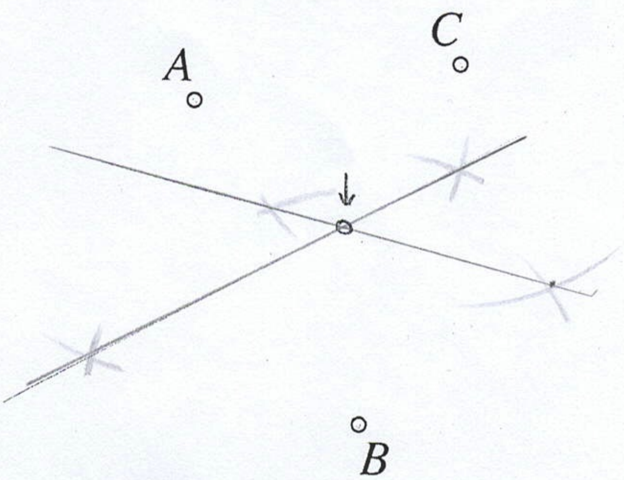
Important: **"Construct" means "construct using an unmarked ruler and a compass". The phrase "unmarked ruler" stands for any ruler that may be used only as a straight edge to draw straight line segments. When you use a compass, show the (intermediate) circular arcs you draw in your constructions (do <u>NOT</u> erase them). Use words to describe BRIEFLY what you have done.**

[7 points] 1. **(a) [4]** Construct a triangle with interior angles of 90°, 30° and 60°.

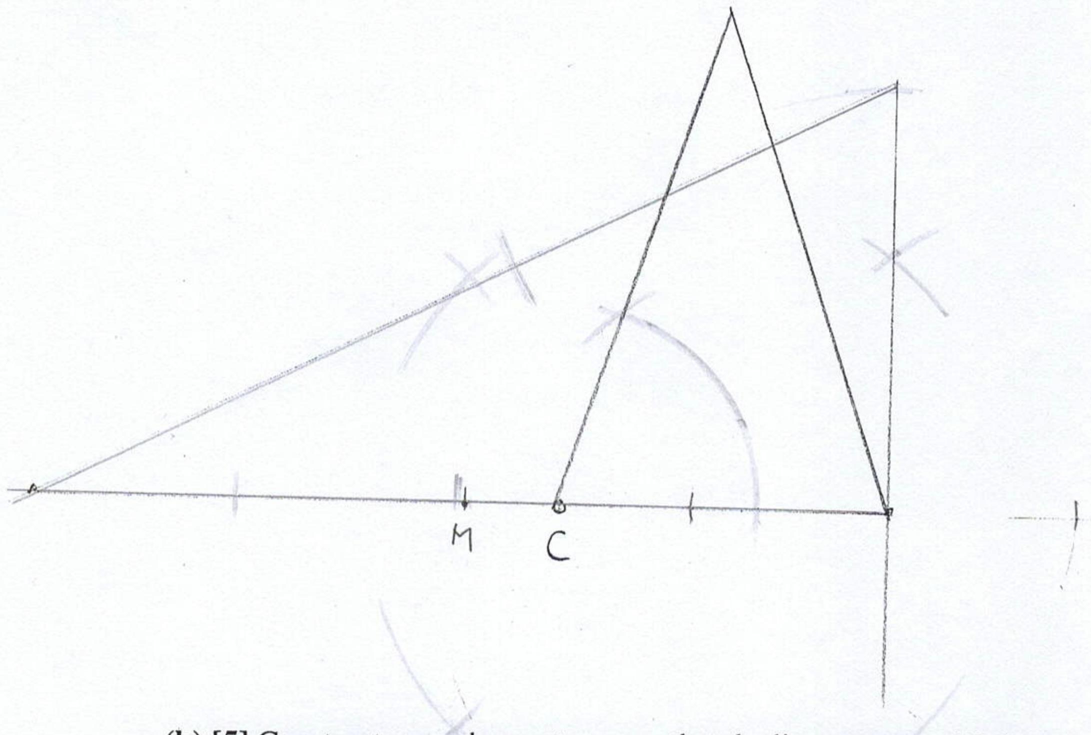


(b) [3] The points A, B and C (shown below) all lie on a single circle. Construct the center of that circle.

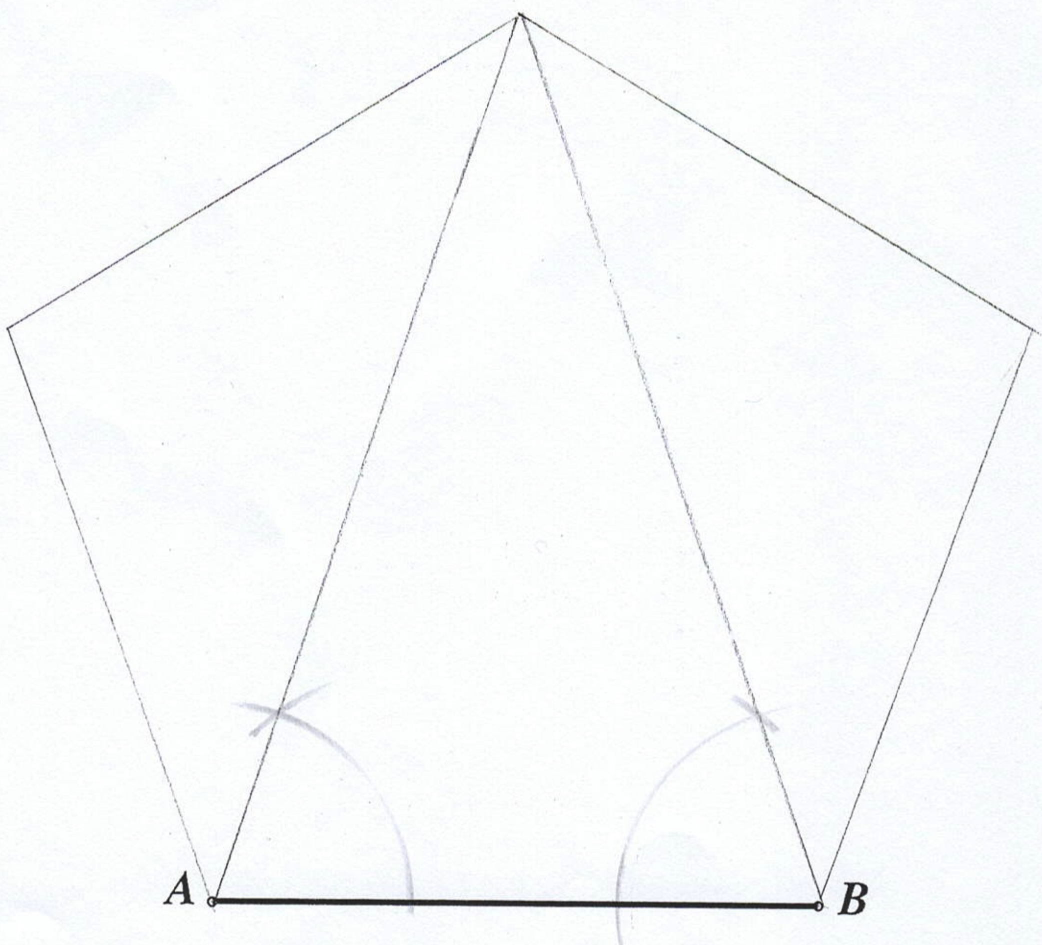


[10 points] 2.

(a) **[5]** Construct any acute golden triangle.



(b) **[5]** Construct a regular pentagon so that the line segment AB shown below is one of its five sides. You may carry over (with a compass) anything constructed in part (a).



[6 points] 3. **(a) [2]** What are Fibonacci numbers? (Write down a precise definition.)

A SEQUENCE OF NUMBERS $f_1, f_2, f_3, \ldots$ SUCH THAT

$f_1 = 1$, $f_2 = 1$, $f_n = f_{n-1} + f_{n-2}$ FOR $n \geq 3$

(b) [4] In this problem f_{20}, f_{21}, f_{22} are Fibonacci numbers. If $2f_{20} = 13530$ and $2f_{22} = 35422$, compute f_{21}. Show your work! (You get no marks if your answer is not justified.)

$f_{22} = f_{21} + f_{20}$

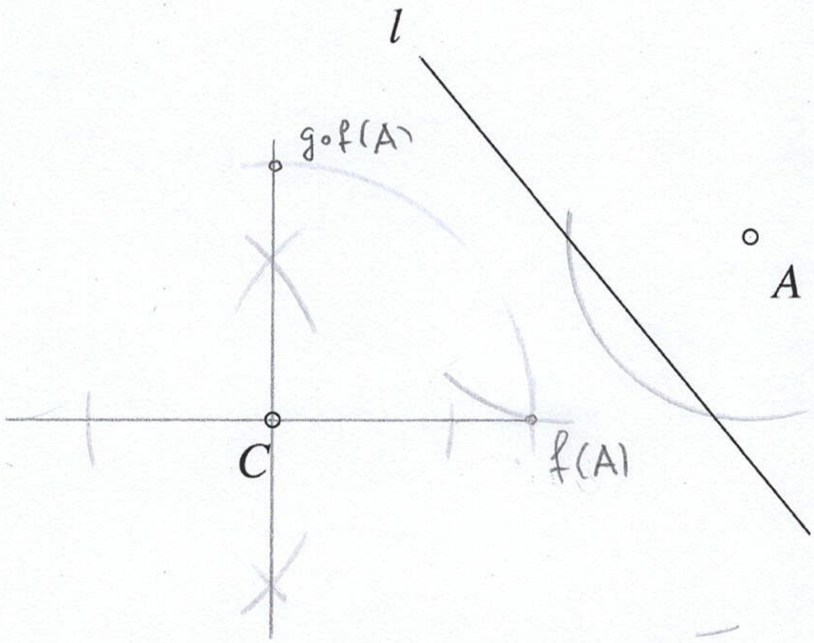
so $2f_{22} = 2f_{21} + 2f_{20}$

so $35422 = 2f_{21} + 13530$

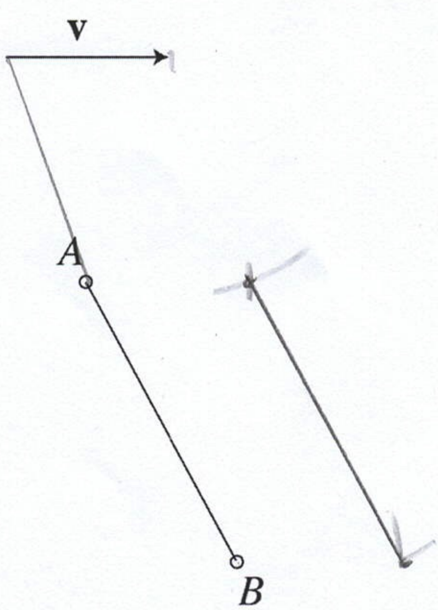
so $2f_{21} = 21892$

so $f_{21} = 10946$

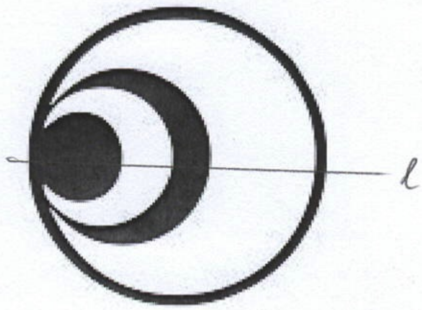
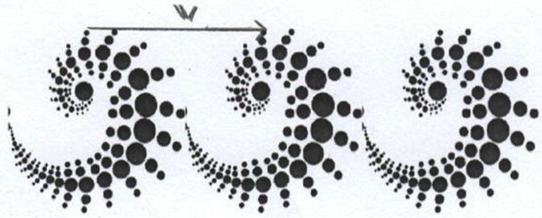
[10 points] 4. [6] (a) Find the image of the point A (shown below) under the composition $g \circ f$ of f followed by g, where f is the reflection with respect to the line l (shown below), and g is the rotation around the center C (shown below) through an angle of 90°.



(b) [4] Construct the image of the line segment AB (shown below) under the translation with respect to the vector $\boldsymbol{v}$ (shown below).

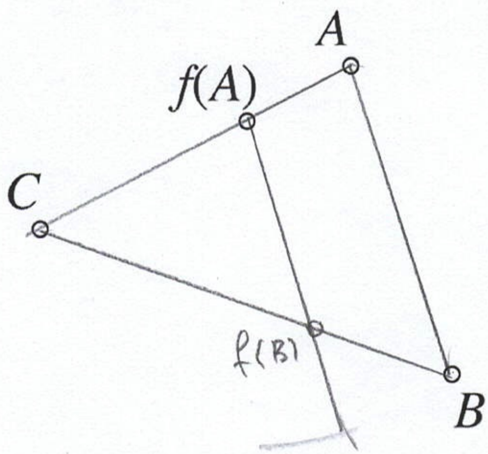


[13 points] 5. Find the group of symmetries of each of the three objects shown below. If you claim a rotational symmetry, indicate the center of the rotation and the angle of rotation. If there are reflections, show the line of reflection. If you use translations, describe the vectors of the translation, drawing **precisely** at least one of them. [The objects are defined by the black points.]

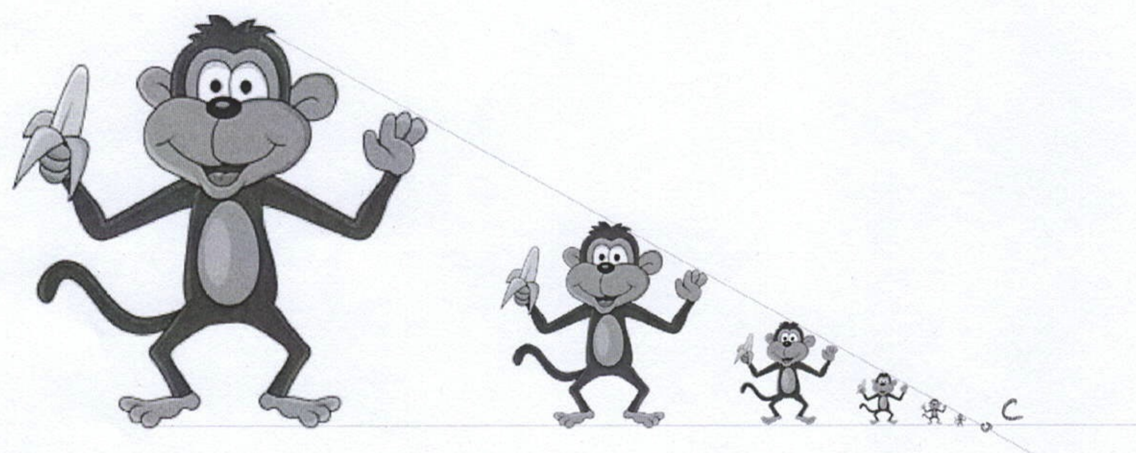
OBJECT	THE GROUP OF SYMMETRIES
	(a) [2] $\{id, ref_\ell\}$
	(b) [5] $\{id, rot(C, 60°), rot(C, 120°)$ $rot(C, 180°), rot(C, 240°)$ $rot(C, 300°)\}$
 ... ... [This is a Frieze pattern and it extends without end both to the left and to the right.]	(c) [6] $\{id, tran_w, tran_{2w}, tran_{3w}, \ldots$ $\ldots, tran_{-w}, tran_{-2w}, \ldots\}$

[9 points] 6.

(a) [5] Let *f* be the central similarity centered at the point *C* (shown below). We are also given the points *A* and *B*, and the point $f(A)$ obtained by applying *f* to *A*. Construct the point $f(B)$ obtained by applying *f* to *B*.



(b) [4] In the figure below every funny monkey except the first one is twice smaller than the funny monkey immediately to the left of it. This pattern of monkeys continues without end, and the final outcome is a fractal called *F*. Confirm that the result is indeed a fractal: find a proper central similarity *f* that sends *F* within itself. You get the full marks if you indicate in the figure the center of your central similarity *f*, and write down the value of the stretching factor α of your central similarity.



$\alpha = 1/2$