# Math 1020 Math in Art <br> Midterm Exam, February 25, 2013 

## Name:

$\qquad$ Student Number: $\qquad$

## CHECK ONE BOX

A01 Michelle Davidson
A02 Sasho Kalajdzievski

| 1. | $\max =13$ |  |
| :--- | :--- | :--- |
| 2. | $\max =14$ |  |
| 3. | $\max =16$ |  |
| 4. | $\max =8$ |  |
| TOTAL | 50 |  |

Important: "Construct" means "construct using an unmarked ruler and a compass". The phrase "unmarked ruler" stands for any ruler that may be used only as a straight edge to draw straight line segments. When you use a compass, show the (intermediate) circular arcs you draw in your constructions (do NOT erase them). Use words to describe BRIEFLY what you have done.
[5 points] 1. (a) Construct the circle that passes through the points $A, B$ and $C$.

## B

A

## ${ }^{\circ} C$

Solution. Construction 5, page 9 in the textbook.
[7 points] 1. (b) Construct the division of the line segment $A B$ (shown below) into three segments of equal length.


Solution. See construction 7, page 11, textbook. Also see class notes since this question was done in class.
[ 9 points] 2. (a) Construct the golden rectangle having the shorter side as the given line segment.

Solution. This is Construction 2, page 17, textbook.
[5 points] (b) Construct a golden obtuse triangle.

Solution. Construct the golden cut $C$ as in Construction 1, page 15, textbook; see the graphics in step 3 in that construction. Then construct an isosceles triangle with base AC, and the other two sides being of size $C B$.
[4+5+7 points] 3. Find the group of symmetries for each of the two objects shown below. If you claim a rotational symmetry, indicate the center of the rotation and the angle of rotation. If there are reflections, show the line of reflection. If there are translational symmetries describe the vectors of translation, drawing precisely at least one of them. [In both (b) and (c) the object is defined by the black points.]

| OBECT | THE GROUP OF SYMMETRIES |
| :---: | :---: |
|  | (a) $\left\{i d, r e f_{l}\right\}$, where $l$ is the horizontal line through the middle of the object (you should have sketch that line in the figure to the left). |
|  | (b) <br> Denote by $O$ the obvious center of the graphics. Then the group of symmetries is $\begin{aligned} & \left\{i d, \operatorname{rot}\left(O, 72^{\circ}\right), \operatorname{rot}\left(O, 144^{\circ}\right),\right. \\ & \left.\operatorname{rot}\left(O, 216^{\circ}\right), \operatorname{rot}\left(O, 288^{\circ}\right)\right\} \end{aligned}$ |
| [This is a Frieze pattern and it extends without end both to the left and to the right. ] | (c) Denote by $\boldsymbol{v}$ the vector starting at the base of the first flower shown in the graphics, to the base of the third flower from the right; this vector should have been drawn precisely with a ruler. Denote by $O_{i}$ the point in the middle between two consecutive flowers, $i=\ldots-3,-2,-1,0,1,2,3, \ldots$; these should also have been shown in the figure. The group is <br> $\left\{\right.$ id, tran $_{v}$, tran $_{2 v}$, tran $_{3 v}, \ldots$ <br> tran $_{-v}$, tran $_{-2 v}$, tran $_{-3 v}, \ldots$ $\ldots, \operatorname{rot}\left(O_{-2}, 180^{\circ}\right), \operatorname{rot}\left(O_{-1}, 180^{\circ}\right)$ <br> $\operatorname{rot}\left(O_{0}, 180^{\circ}\right), \operatorname{rot}\left(O_{1}, 180^{\circ}\right) .$, <br> $\left.\operatorname{rot}\left(O_{2}, 180^{\circ}\right), \ldots\right\}$ |

[4+4 points] 4. The point $f(A)$ is obtained from the point $A$ by applying a central similarity $f$ of a stretching factor $a=\frac{1}{2}$.
(a) Construct the center $O$ of the central similarity $f$.
(b) Construct the image $f(C D)$ of the line segment $C D$ under the central similarity $f$.


$$
\begin{array}{cc}
\circ & \circ \\
f(A) & A
\end{array}
$$

## Solution.


(a) Prolong the line between $f(A)$ and $A$ to the left, then mark the point $O$ so that the distance from $O$ to $f(A)$ is the distance from $f(A)$ to $A$.
(b) Find the midpoint of $O C$; that is $f(C)$. Find the midpoint of $O B$; that is $f(B)$. Join $f(C)$ and $f(B)$.

