Math 1020 Math in Art
Midterm Exam, March 1, 2011

Name: ______________________________ **Student Number:** ______________

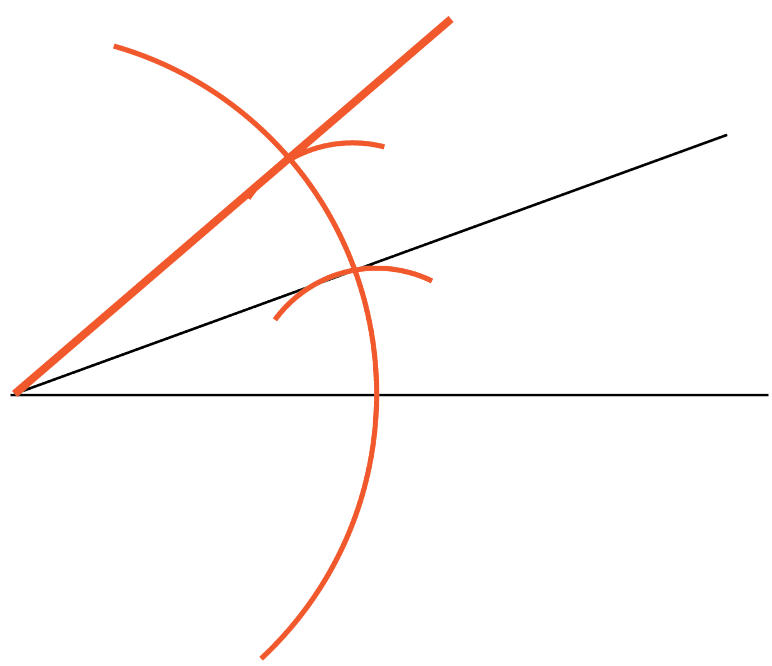
1.	*max=8*	
2.	*max=10*	
3.	*max=10*	
4.	*max=12*	
5.	*max=10*	

Total=50

Important: **"Construct" means "construct using an unmarked ruler and a compass". The phrase "unmarked ruler" stands for any ruler that may be used only as a straight edge to draw straight line segments. When you use a compass, show the (intermediate) circular arcs you draw in your constructions (do NOT erase them). Use words to describe BRIEFLY what you have done.**

[8 points] 1. **(a)** In the picture below we show an angle of 20º. Construct an angle of 40º.

Solution. **As indicated.**



(b) Costruct a square such that its sides are three times shorter than the line segment *AB* shown below.

Solution. **First divide the line segment into three equal parts; this is done both in class and in the textbook. Then construct a square with its side being one of these smaller parts**

[10 points] **2.** **(a) [7]** Recall that in a golden rectangle the ratio of the base with respect to the height is the golden section ϕ. Construct a rectangle with its base equal to the given line segment AB, and the ratio of its base over its height equal to $\frac{\phi}{2}$.

Solution: **Construct the golden cut C (as in the textbook or the notes), and use the length of the line segment AC as the height. This gives the golden rectangle. Then find the midpoint of the height of the golden rectangle and construct a rectangle twice shorter.**

(b) [3] Construct an obtuse golden triangle of the line segment *AB* below (of the same length as the one used in part (a)!)

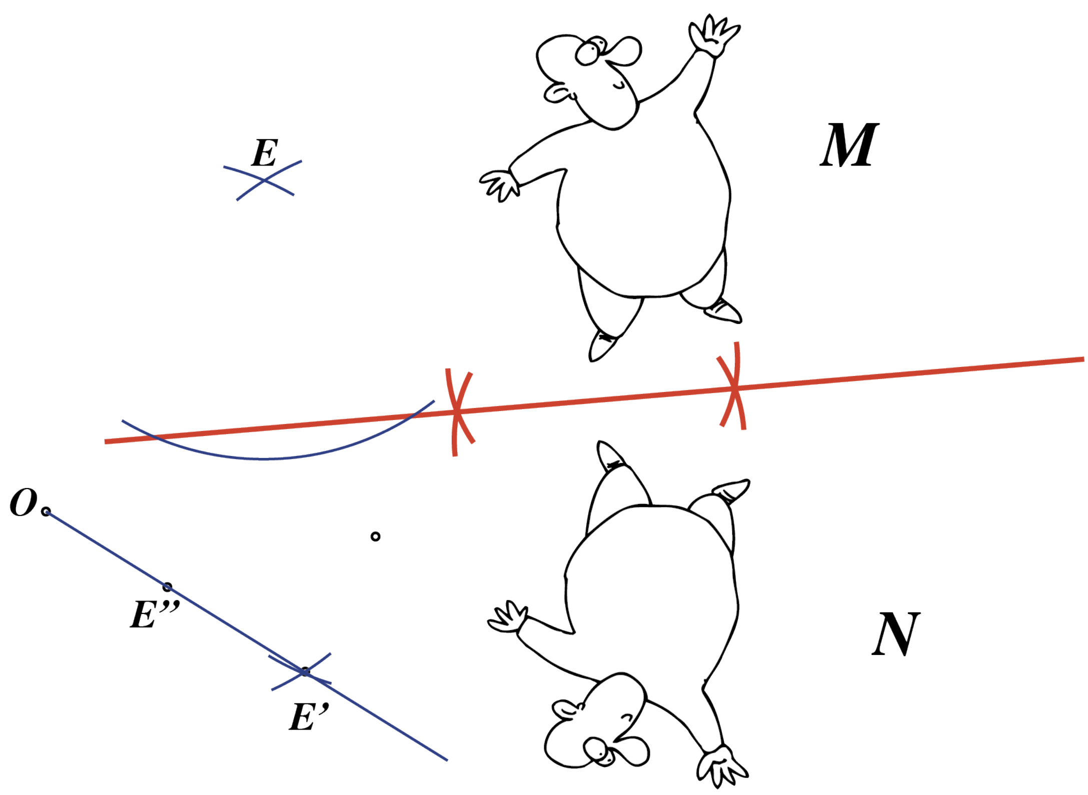
Solution. **Use AC from part (a) as the length of the two other sides of the triangle.**

[10 points] **3.** **(a)** **[4]** The object N is obtained from the object M by reflecting it with respect to a line l. Construct the line l.

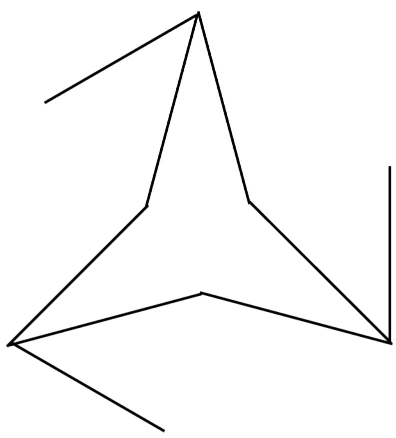
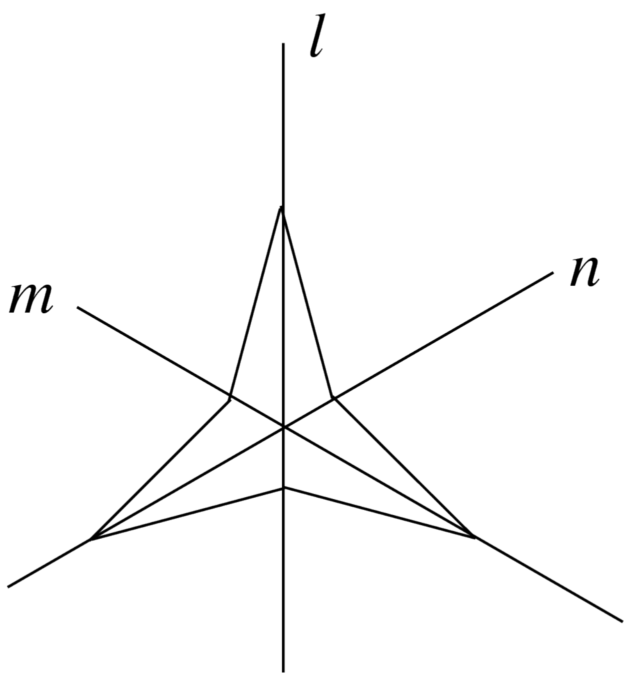
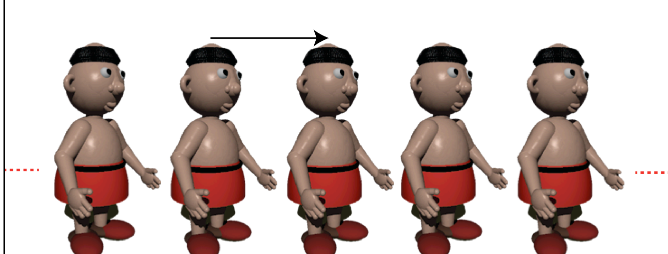
(b) **[6]** Construct the image of the point E shown below under the composition of the reflection with respect the line l (as constructed in part (a)), followed by the central similarity centered at the point O as shown below and of stretching factor $\alpha = \frac{1}{2}$.

***Solution*. (a) The line of reflection is in red (obtained by bisecting any line segment connecting two corresponding points of the figures M and N)**

(b) First reflect E with respect to the red line to get E'; then find the midpoint E'' of the line segment OE'.

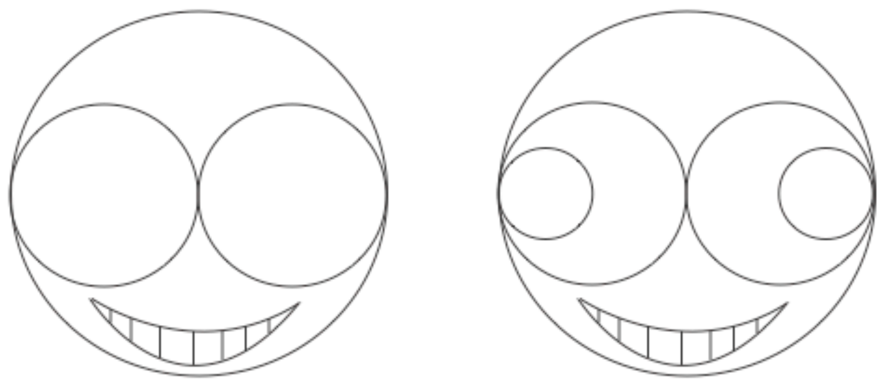


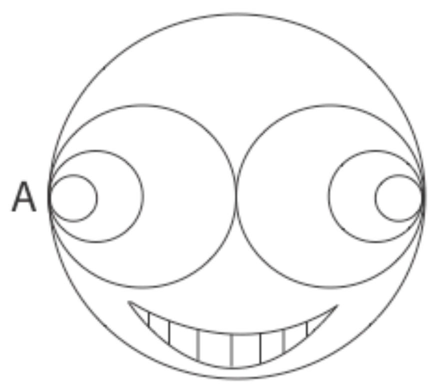
[12 points] **4. (a)** Draw an object and a point O in the empty square (a) below so that its group of symmetries is $\{id, rot(O,120), rot(O,240)\}$. Find the group of symmetries for each of the two objects shown below. If you claim a rotational symmetry, indicate the center of the rotation and the angle of rotation. If there are reflections, show the line of reflection. If there are translational symmetries describe the vectors of translation, drawing **precisely** at least one of them. [In both (b) and (c) the object is defined by the black points.]

OBECT	THE GROUP OF SYMMETRIES
(a) Here is one such object (out of many) 	$\{id, rot(O,120), rot(O,240)\}$
	(b) $\{id, rot(O,120), rot(O,240), ref_l, ref_m, ref_n\}$ where l, n, and m are the lines as shown, and O is their common intersection point (in the center of the object)
 [This is a Frieze pattern and it extends without end both to the left and to the right.]	(c) $\{id, trans_{\mathbf{v}}, trans_{2\mathbf{v}}, trans_{3\mathbf{v}}, \ldots$ $\ldots, trans_{-\mathbf{v}}, trans_{-2\mathbf{v}}, trans_{-3\mathbf{v}}, \ldots\}$ where **v** is the vector shown to the right.

[10 points] 5. In the two figures we show the first two steps in the construction of a fractal. Note that in the second figure the point *B* is the midpoint of the segment AC.

(a) Draw with a ruler and a compass the figure representing the next step in the construction of the fractal. (Points will be subtracted for ugly drawings!)





(b) The final fractal **F** is constructed after repeating the above steps infinitely many times. Describe two central similarities of stretching factors not equal to 1 that will send **a part of** the fractal **F** into itself. (To get full marks here, you need to indicate in the drawing you do in part (a) where exactly the centers of the central similarities are, and you need to state a specific numbers for the stretching factors of that central similarities.)

Solution. **(a) Shown above (where the third step is the figure in the second row from above.**

(b) The central similarity centered at A and of stretching factor $\frac{1}{2}$ **sends the left 'eye' within itself.**