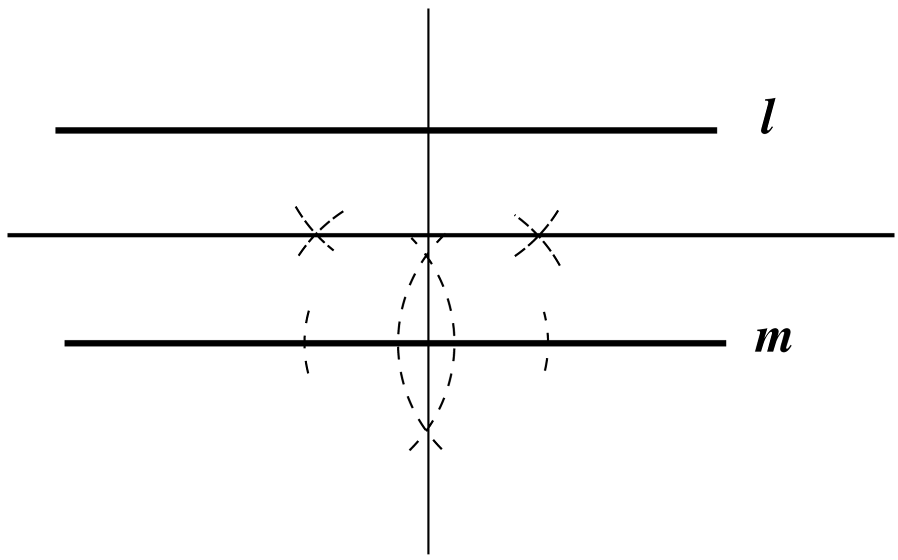
Math 1020 Math in Art
Midterm Exam, February 25, 2010

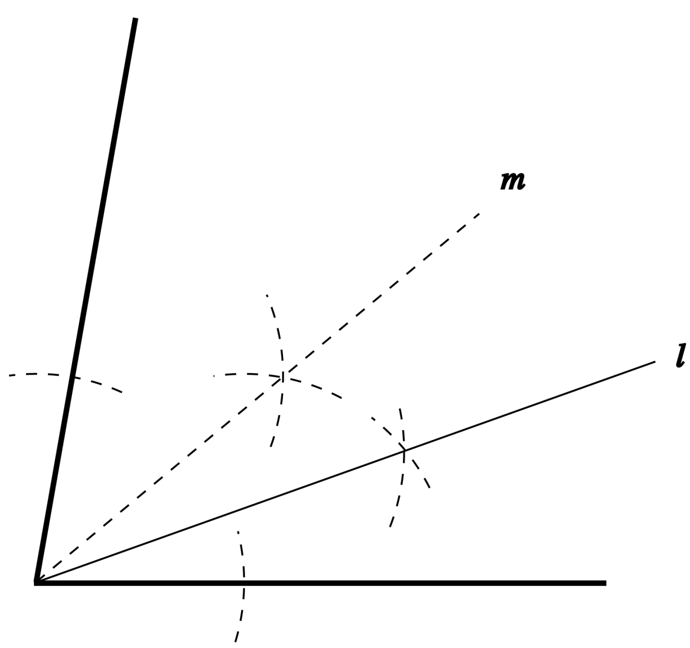
Brief Solutions

[8 points] 1. **(a)** The lines *l* and *m* shown below are parallel. Construct any line that is parallel to both *l* and *m*.



Solution. Construct a perpendicular to *m* at any point (the outline of the construction shown in the figure; details given in the book or in the class-notes); It is at the same time perpendicular to *l*. Then repeat this construction to get the parallel line (done in class; the construction is shown in the above graphics).

(b) In the picture below we show an angle of 80º. Construct an angle of 20º.



Solution. First bisect the given angle to get *m* and an angle of 40º. (The outline of the construction shown in the figure; details given in the book or in the class-notes). Then repeat the same construction to subdivide the angle of 40º into two equal angles, each of which will be 20º.

[10 points] **2.** **(a)** Find the golden cut *C* of the line segment *AB* shown below (with *C* closer to *B* than to *A*), then construct an acute golden triangle with the base of that triangle being the line segment *CB*.

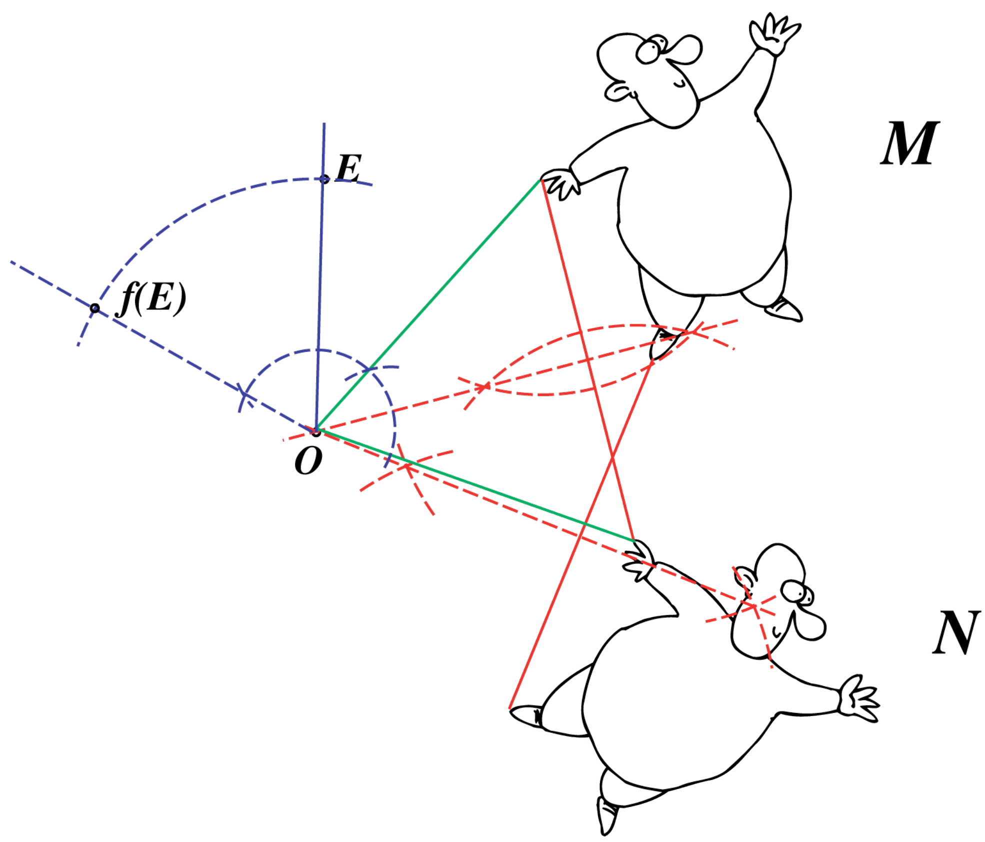
Solution. This is done in class and in the book.

(b) Construct a regular pentagon with one side being the line segment *DE* shown below. (Note and hint: You may use the acute golden triangle constructed in part (a) in the construction of the regular pentagon.)

Solution. This is also done in class and in the book.

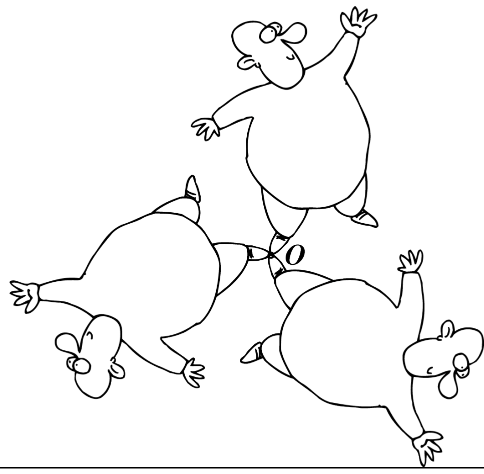
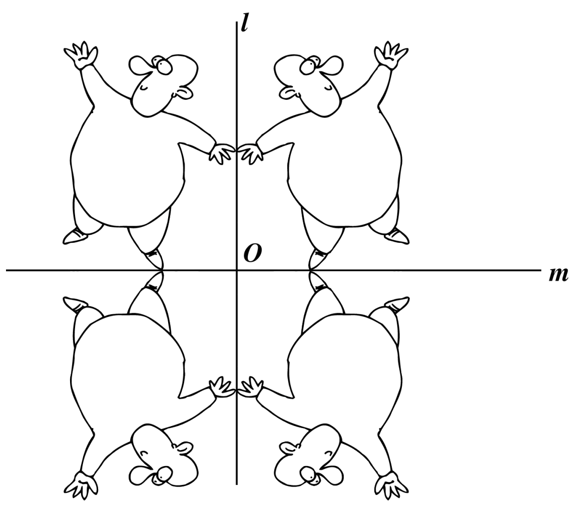
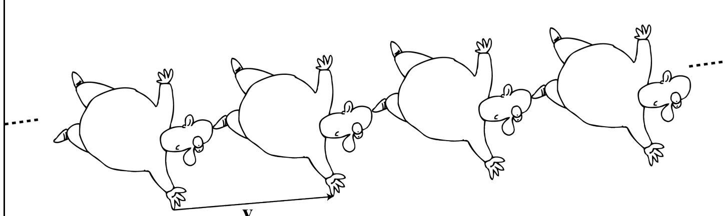
[10 points] **3.** **(a)** The object M is obtained from the object N by rotating the latter around a point *O* and through an angle α . Construct the center of the rotation *O* and an angle of the rotation.

(b) Find the image of the point *E* shown below under the rotation defined in part (a) of this question. (Hint: you would need to duplicate an angle here!)



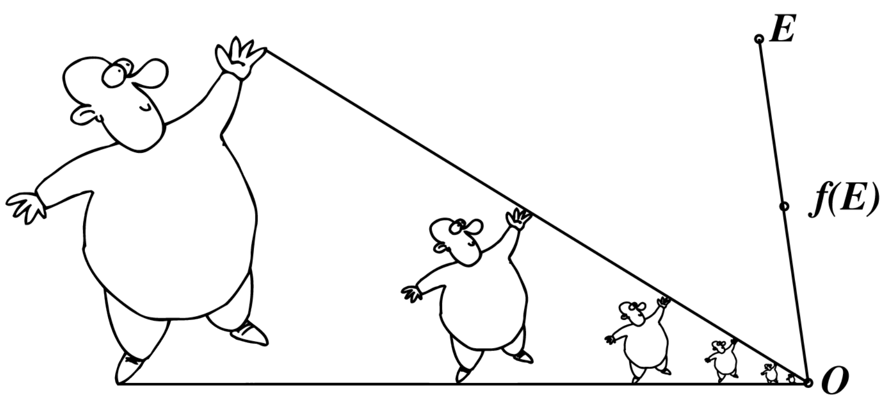
Solution. This is Example 6, page 39 in the textbook. The construction of the center *O* of the rotation is shown in red. The angle of the rotation is marked in green. This angle is then duplicated over the ray *OE*, and the point *E* is then rotated through that angle to get the image denoted by *f(E)* (the last part of the construction is shown in blue.)

[12 points] **4.** Find the group of symmetries for each of the three objects shown below. If you claim a rotational symmetry, indicate the center of the rotation and the angle of rotation. If there are reflections, show the line of reflection. If there are translational symmetries describe the vectors of translation, drawing **precisely** at least one of them. [In all three cases the object is defined by the black points.]

OBECT	THE GROUP OF SYMMETRIES
	{identity, rot(O, 120°), rot(O, 240°)} where the center *O* of the rotations is shown to the left.
	{identity, rot(O, 180°), ref_l . ref_m} where the lines *l* and *m*, as well as the center *O* are shown to the left.
 [This is a Frieze pattern and it extends without end both to the left and to the right.]	{identity, $tran_{\mathbf{v}}, tran_{2\mathbf{v}}, tran_{3\mathbf{v}}, ...,$ $tran_{-\mathbf{v}}, tran_{-2\mathbf{v}}, tran_{-3\mathbf{v}}, ...$} where the basic vector **v** of the translation is indicated on the picture to the left.

[10 points] 5. (a) The object shown below (except for the point E given to the right and above it; ignore *E* in this part) shows an infinite sequence of cartoon-like characters, each twice smaller than the one immediately preceding it (from left to right). Show that this is a complete fractal by describing a central similarity *f* that sends this object within itself. (Note: to get the full mark here you only need to indicate the position of the center of *f* and to write down the number that is the stretching factor of *f*.)

(b) Using the central similarity *f* that you have described in part (a) of this question, construct the image $f(E)$ of the point *E* shown below.



Solution. (a) the central similarity centered *f* at *O* (as shown) and of stretching factor $\alpha = \frac{1}{2}$ maps the object within itself (thus showing it is a complete fractal).

(b) The image *f(E)* of the point *E* under the central similarity described in part (a) is at the midpoint of the line segment *OE*. Construction that midpoint was the very first construction in this course.