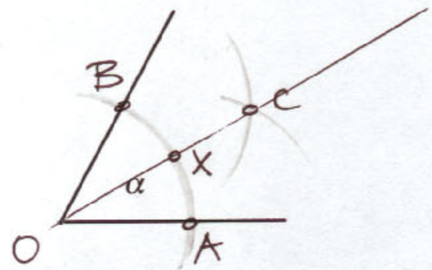
to draw straight line segments. When you use a compass, show the (intermediate) circular arcs you draw in your constructions (do not erase them – they justify the steps you have used in the constructions and I consider them when marking).

1. **(a) [5]** Construct an angle β that is twice smaller than the angle α shown below.



1. CIRCLE AT O, ANY RADIUS. GET A AND B
2. TWO INTERSECTING CIRCLES AT A AND B. GET C
3. CONNECT OC.

isosceles

(b)[4] Construct an ~~equilateral~~ triangle over the given base (see below) with both of the angles at the base equal to β (with β constructed in part (a)).

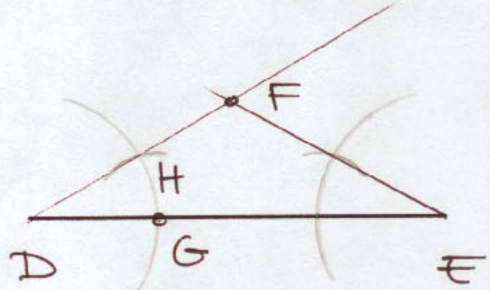
(c)[3] Assume in this part that β is 35°. What is the size of the third angle in the ~~equilateral~~ triangle constructed in part (b)?

isosceles

REPLICATE THE ANGLE $\measuredangle$ AOX (SEE PART a) OVER DE AND ED.

- CIRCLE AT D RADIUS OA; GET G
- CIRCLE AT G RADIUS AX; GET H
- CONNECT DH
- SAME OVER E

GET Δ DEF AS NEEDED.

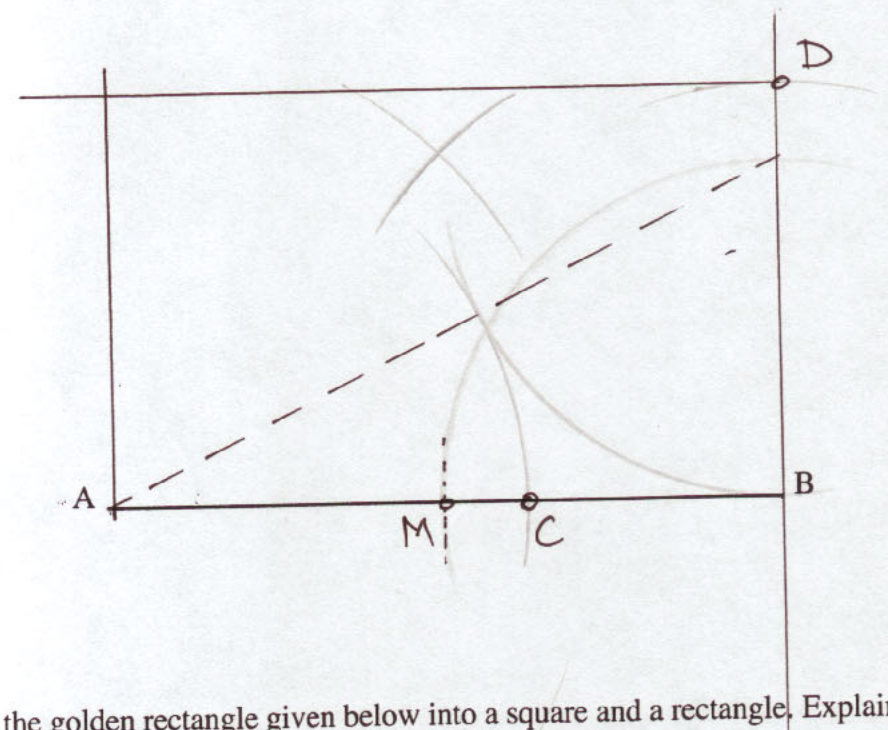


THE ANGLE AT F is $180° - (2)(35°) = 110°$.

2. **(a)[6]** Construct a golden rectangle over the line segment given below (the longer side of the rectangle). You may **not** use the second picture below (the depicted golden rectangle). Do not forget to briefly describe your steps.

1. FIND THE GOLDEN CUT OF AB (AS IN CLASS). DENOTE IT BY C. THEN CONSTRUCT A RECTANGLE OVER AB WITH HEIGHT ~~B~~ AC

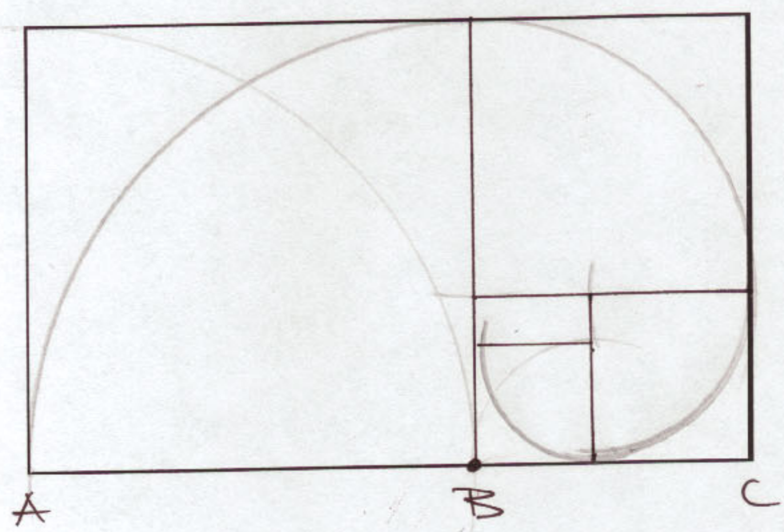
BD = AC



(b)[3] Subdivide the golden rectangle given below into a square and a rectangle. Explain why the smaller rectangle that you get must be golden?

(c)[3] Keep subdividing the (smaller) golden rectangles into squares and golden rectangles until you get 4 (smaller and smaller) squares. Construct a (part of a) golden spiral using that subdivision.

SINCE $\frac{AC}{AB} = \phi$ = GOLDEN RATIO IT FOLLOWS THAT $\frac{AB}{AC} = \phi$ TOO (BY DEFINITION OF ϕ). THIS ANSWERS THE QUESTION IN PART (b).

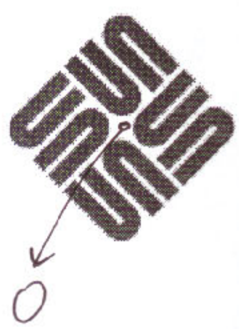


THE (BEST OF THE) CONSTRUCTION IS CLEAR FROM THE PICTURE.

3. List (without justification) all of the symmetries of the objects ***A*** and ***B*** below.

(Describe clearly the symmetries that you list. For example, if you list a rotation as a symmetry then say "a rotation centered at O and for an angle of x many degrees", where O is a point you should identify in the picture, and x is a specific angle measure you should find.)

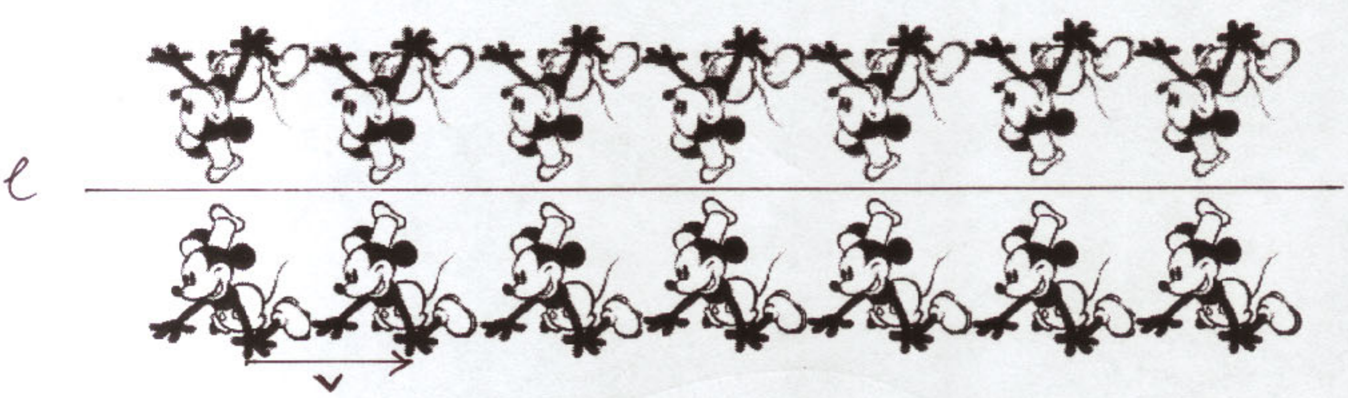
(a)[5.5] Object A



SYMMETRY GROUP =

{IDENTITY, ROTATION ABOUT O FOR 90°, ROTATION ABOUT O FOR 180°, ROTATION ABOUT O FOR 270°}.

(b)[7.5] Object B (Assume that this pattern extends unboundedly to the left and to the right. Ignore minor details in the shapes and positions of the copies of Mickey.)



SYMMETRY GROUP:

(1) identity

(2) REFLECTION WITH RESPCET TO THE LINE l

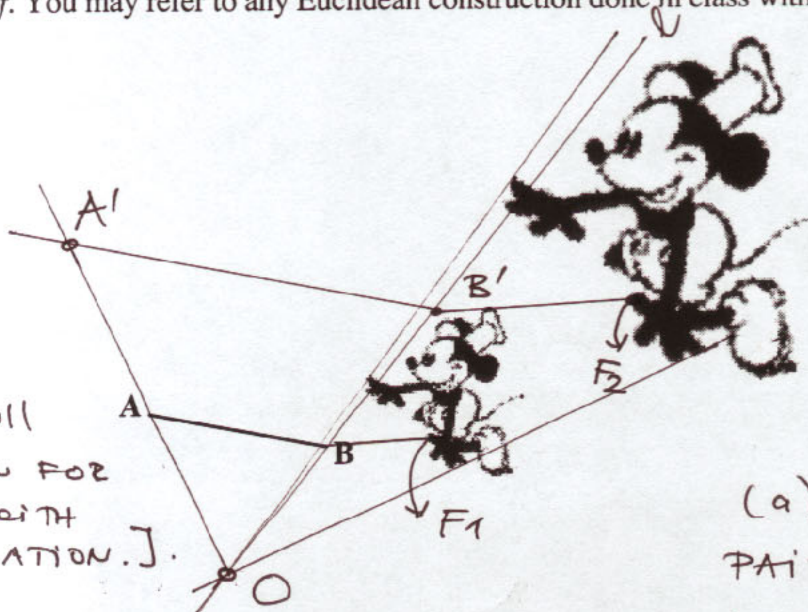
(3) TRANSLATIONS ~~[illegible]~~ ALONG V, 2V, 3V, ... (INFINITELY MANY)
TRANSLATIONS ALONG −V, −2V, −3V, ... (INFINITELY MANY).

COMPOSITION OF ANY TRANSLATION IN (3) FOLLOWED BY THE REFLECTION IN (2) — GLIDE REFLECTIONS (INFINITELY MANY).

4. The larger Mickey (see the picture below) is obtained from the smaller Mickey by applying to the latter a central similarity *f*.

(a)[5.5] Construct the center of the similarity *f*.

(b)[7.5] Construct the image of the line segment AB (see the picture below) under the similarity *f*. You may refer to any Euclidean construction done in class without getting into details.



NOTE FOR (b): FULL ARKS WERE GIVEN FOR ORRECT PICTURE WITH SHORTER EXPLANATION.].

(b). CONNECT A POINT IN THE SMALLER COPY OF MICKEY TO B. (POINT F_1 IN THE PICTURE). DRAW A LINE THROUGH THE CORRESPONDING POINT F_2 AND PARALLEL TO BF_1. THAT LINE INTERSECT THE LINE l THROUGH OB AT B'. THIS B' IS THE IMAGE OF B UNDER f. FINALLY DRAW A LINE THROUGH B' PARALLEL TO AB AND FIND THE INTERSECTION WITH THE LINE OA TO GET A'

(a) CONNECT A PAIR OF CORRESPONDING POINTS ON THE TWO COPIES OF THE MOUSE. REPEAT THIS FOR ANOTHER PAIR OF CORRESPONDING POINTS. THE INTERSECTION OF THE TWO LINES GIVES THE CENTER O OF THE SIMILARITY f.

5. [BONUS 5] True or false? Do **not** justify anything in this question.

FALSE (a) It is possible to subdivide any given angle into three smaller equal angles by means of a Euclidean construction (with an unmarked ruler and a compass).

FALSE (b) In an acute golden triangle the ratio of the length of the base by the length of one of the other two sides is the golden ratio.

TRUE (c) For any three consecutive Fibonacci number the largest of them is the sum of the other two.

TRUE (d) The composition of the rotation around a point O by 45° followed by the rotation around the same point by 55° is the rotation around O by 100°.

TRUE (e) Any central similarity with stretching factor equal to 1 is the same as the identity symmetry.