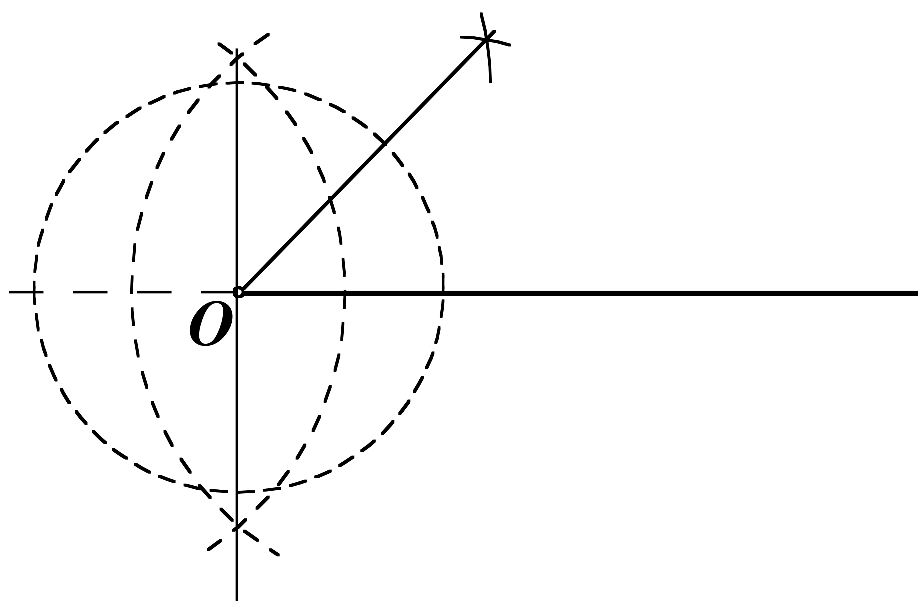
Math 1020 Math in Art
Midterm Exam
February 22, 2007
<u>Brief</u> Solutions

1. Construct (using an unmarked ruler and a compass) an angle of 45° with a corner at O and over the given semi-line. Do not forget to briefly describe your steps! (Hint: first construct an angle of 90°.)

Solution. First construct a perpendicular to the given line segment at the point *O*, then subdivide thus constructed right angle into two equal angles (each of them of 45°).



2. (a) Construct an acute golden triangle over the given line segment.

Solution. First construct any acute golden triangle. I am not going to show the construction here since it is given both in the lectures and in the lecture notes. Then duplicate the basis angles of thus constructed golden triangle over the given line segment. The whole construction is given on pages 15-16 in the lecture notes.

2. (b) Construct an a regular pentagon over the given side.

Solution. It is given on page 19 of the lecture notes.

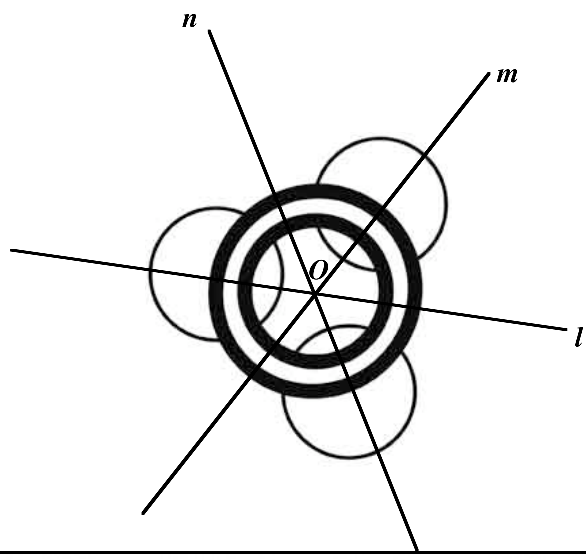
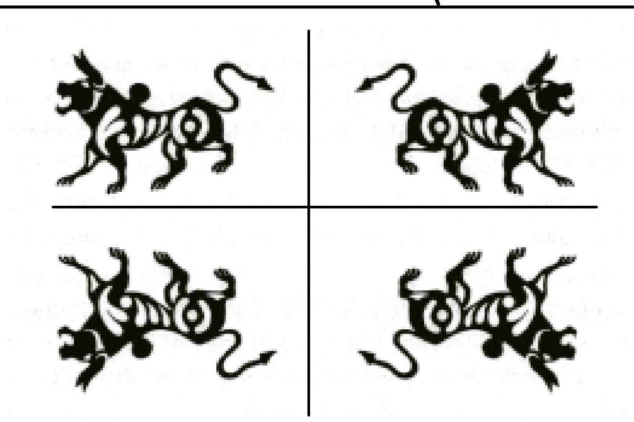
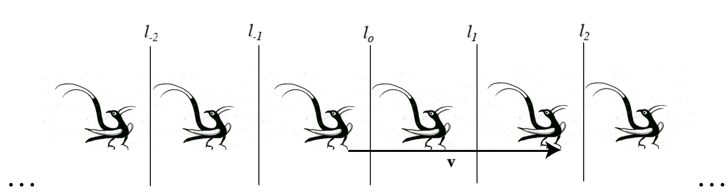
3. (a) What are Fibonacci numbers? (Write a precise definition.)

Solution. Page 20 in the lecture notes.

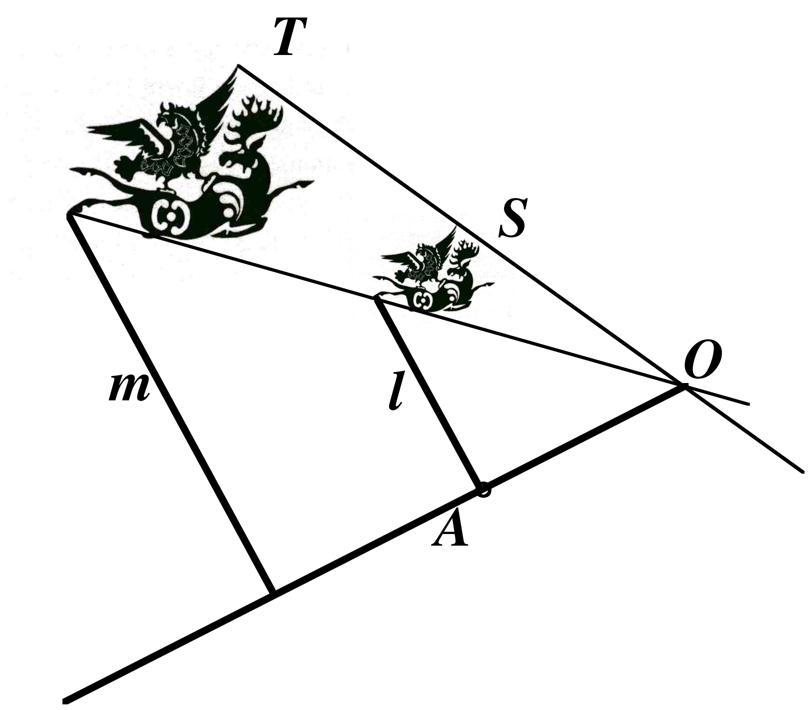
(b) The 22nd Fibonacci number f_{22} is 17711. The 24th Fibonacci number f_{24} is 46368. Find the 23rd Fibonacci number f_{23}.

Solution.
Since $f_{24} = f_{23} + f_{22}$, we have that $f_{23} = f_{24} - f_{22} = 46368\text{-}17711$.

4. Find the group of symmetries for each of the three objects shown below. If you claim a rotational symmetry, indicate the center of the rotation and the angle of rotation. If there are reflections, show the line of reflection. If there are translational symmetries show or describe the vectors of translation.
(The first diagram represents a crop circle. In the second and the third graphics we use tattoos preserved on a 3000-years-old Altai mummy.)

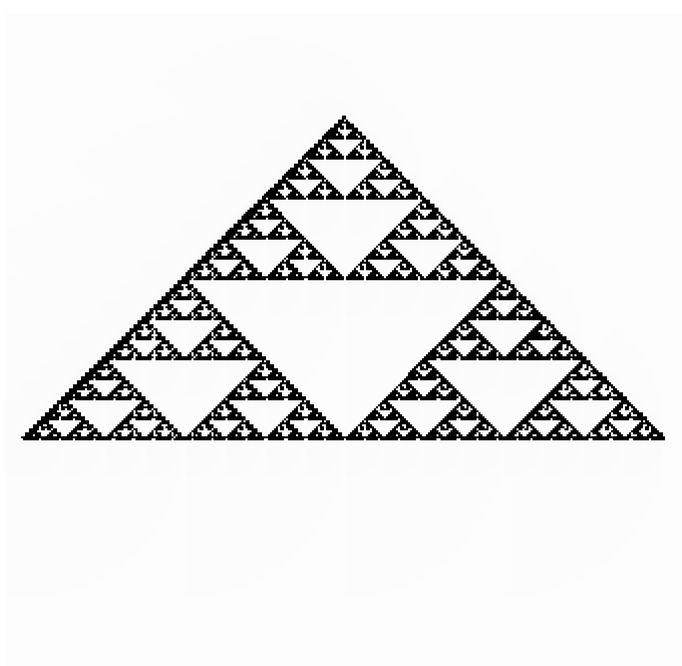
OBECT	THE GROUP OF SYMMETRIES
	{idenitity, $ref(l)$, $ref(m)$, $ref(n)$, rot(O,120º), rot(O,240º)}
	{identity, reflection with respect to the horizontal line shown in the picture, reflection with respect to the vertical line shown in the picture, rotation through 180º around the intersecting point of these two lines}
 [This is a Frieze pattern (the pattern extends without bounds both to the left and to the right).	{identity, $trans(v)$, $trans(2v)$,…., $trans(-v)$, $trans(-2v)$,…, $ref(l_0)$, $ref(l_1)$, $ref(l_2)$,…, $ref(l_{-1})$, $ref(l_{-2})$,…}

5. (a) The object T is the image of the object S under an unknown central similarity *f*. Find the center of the central similarity *f*, and construct the image $f(A)$ of the point *A* under the central similarity *f*.
(We are again using a tattoo of the ancient mummy from Exercise 4.)



Solution. First join two pairs of corresponding points (tips of the wings, and tips of the hind legs, say) to get two lines intersecting at *O*. That is the center of the similarity. Then join O and A; the point $f(A)$ is somewhere on that line. Now join A with any point on the smaller object (get the line *l*), and then construct (this needs some work; done in the book and in the lectures) the line *m* that passes through the corresponding point of the larger object and is parallel to *l*. The desired point $f(A)$ happens at the intersection of *m* and the line through O and A.

(b) The objects shown below is a fractal called Sierpinski triangle. As is visible from the graphics, the Sierpinski triangle is obtained by removing the triangle obtained by connecting the midpoints of the sides of the largest triangle, and then repeating that procedure ad infinitum (infinitely many times) to the smaller triangles we get in each step. Describe one central similarity of stretching factor not equal to 1, which moves the points of the Sierpinski triangle within itself. (In order to describe a central similarity you would need to identify its center and its stretching factor.)



Any corner is good enough for the center of the central similarity, and its stretching factor could be taken to be any of the numbers $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$