THE UNIVERSITY OF MANITOBA

NAME: (PRINT):_____

SIGNATURE (In Ink):_____
(I understand that cheating is a serious offense)

STUDENT NUMBER (In Ink):_____

| DO NOT WRITE IN THIS COLUMN |
| --- |
| 1. _____ / 8 |
| 2. _____ / 9 |
| 3. _____ / 8 |
| 4. _____ / 8 |
| 5. _____ / 9 |
| 6. _____ /10 |
| 7. _____ / 8 |
| 8. Bonus _____ / 5 |
| TOTAL _____ /60 |

Final Exam

Show your work. Leave clear sketches of the intermediate steps you make in the construction. Clarity in your pictures will be appreciated and may affect the marking. Accompany your constructions with short descriptions of the steps you make.

1 [8]. (a) The arc **a** and the point **A** are all parts of an invisible circle. Construct (using an unmarked ruler and a compass) the center of that circle and then draw the rest of the circle.
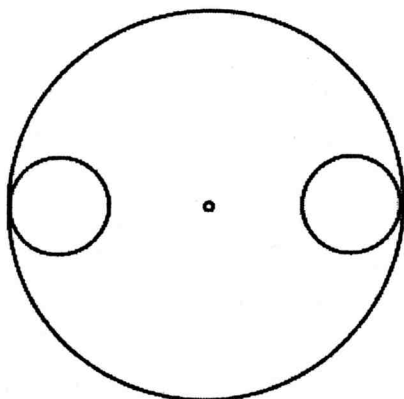
A ₒ

a

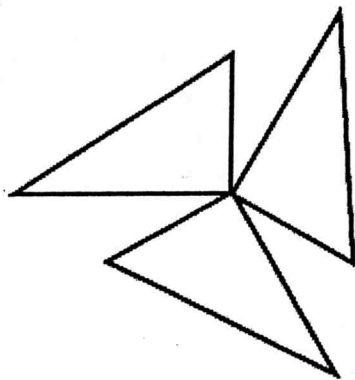(b) Subdivide the circle shown below into six arcs of equal size.
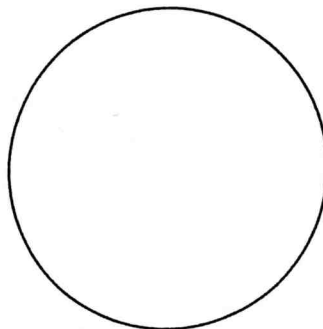
2.[9] Find the symmetries of the following objects.
[If you list a rotation as a symmetry then identify the center and the angle of rotation. If you list a reflection as a symmetry then identify the line of reflection.]

(a) The small circle in the middle indicates the center of the large circle.

(b)

3. [8] **(a)** Construct an acute golden triangle over the given side (see below) .
(b) Use the golden triangle in (a) to construct a regular pentagon over the same side

_____

4. [8] The picture below shows **a part** of a fractal F. The following is known about F: the rest of the fractal is within the smaller square and the fractal F is self-similar with respect to the similarity centered at the corner point O and with stretching factor 1/2. Draw (and explain in one sentence) the rest of the fractal F.

O
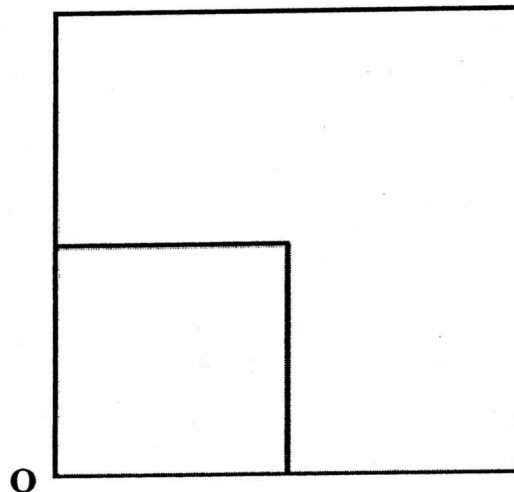
5.[9] (a) What kind of a polyhedron is a platonic solid? (Explain in one sentence.)
How many types of platonic solid are there?

(b) A cube is one type of a platonic solid. The following picture depicts the edges in one corner of a cube and the (dashed) horizon line of the perspective drawing. Draw the rest of the cube.
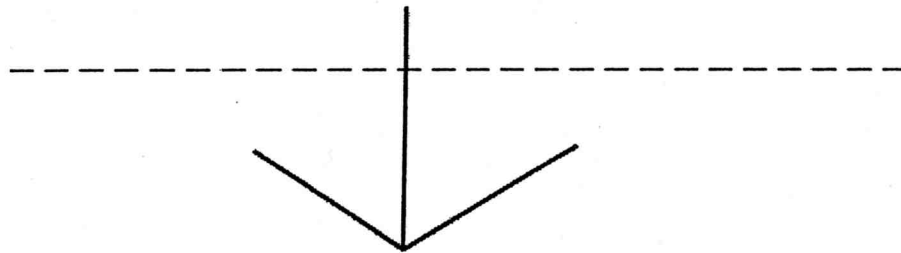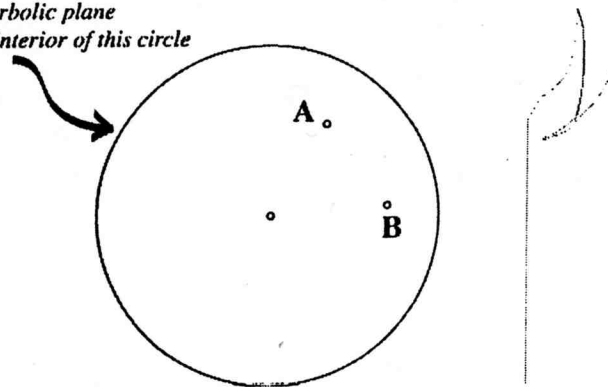
6. [10]

(a) Construct one (any) hyperbolic line that is perpendicular to the given hyperbolic line
l.

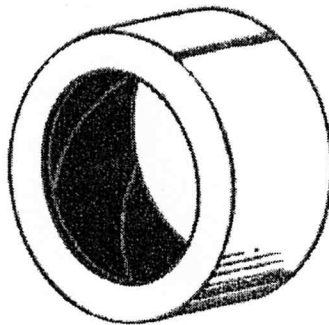*The hyperbolic plane
is in the interior of this circle*
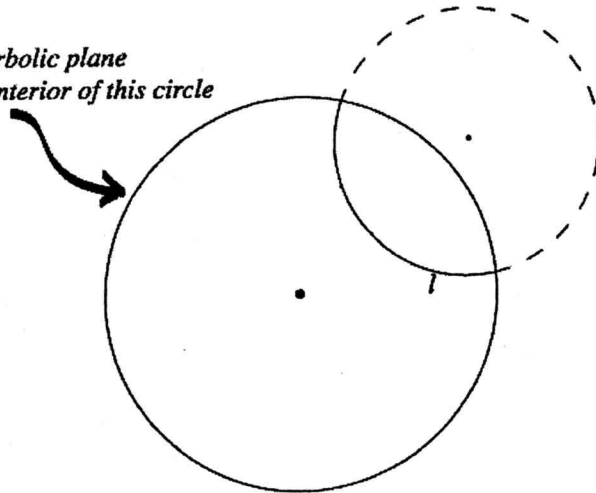
(b) Construct the hyperbolic line passing through the given points A and B.

*The hyperbolic plane
is in the interior of this circle*

A

B

7.[8] Consider the surface of the following tape (**the surface of the tape only**).

(a) Find the genus of the surface of the tape. Briefly justify your answer.

(b) Use (a) to find the Euler's characteristic of the surface of the tape.

8. [5 bonus] True or false? (No need to justify your answer.)

(a) Every golden rectangle can be subdivided into exactly two squares.

(b) The limit of the ratio $\dfrac{f_n}{f_{n+1}}$ is the golden section ϕ.

(c) A composition of any two reflections with respect to a single line is the identity.

(d) If f is a similarity of stretching factor 2 and centered at a point A, then the Julia set associated to f is the point A.

(e) Every two bounded and orientable surfaces (= 2-manifolds) of the same Euler characteristic are homotopic.