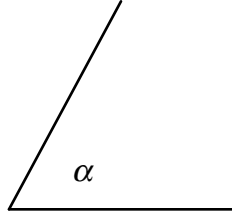


**136.102 Math in Art**  
**Midterm Exam**  
**October 16 2003**  
**(70 minutes)**

*Important:* “Construct” means “construct using an unmarked ruler and a compass”. The phrase “unmarked ruler” stands for any ruler that may be used only as a straight edge to draw straight line segments. When you use a compass, show the (intermediate) circular arcs you draw in your constructions (do not erase them – they justify the steps you have used in the constructions and I consider them when marking).

1. (a) [5] Construct an angle  $\beta$  that is twice smaller than the angle  $\alpha$  shown below.



(b)[4] Construct an equilateral triangle over the given base (see below) with both of the angles at the base equal to  $\beta$  (with  $\beta$  constructed in part (a)).

(c)[3] Assume in this part that  $\beta$  is  $35^\circ$ . What is the size of the third angle in the equilateral triangle constructed in part (b)?



2. (a)[6] Construct a golden rectangle over the line segment given below (the longer side of the rectangle). You may **not** use the second picture below (the depicted golden rectangle). Do not forget to briefly describe your steps.



(b)[3] Subdivide the golden rectangle given below into a square and a rectangle. Explain why the smaller rectangle that you get must be golden?

(c)[3] Keep subdividing the (smaller) golden rectangles into squares and golden rectangles until you get 4 (smaller and smaller) squares. Construct a (part of a) golden spiral using that subdivision.



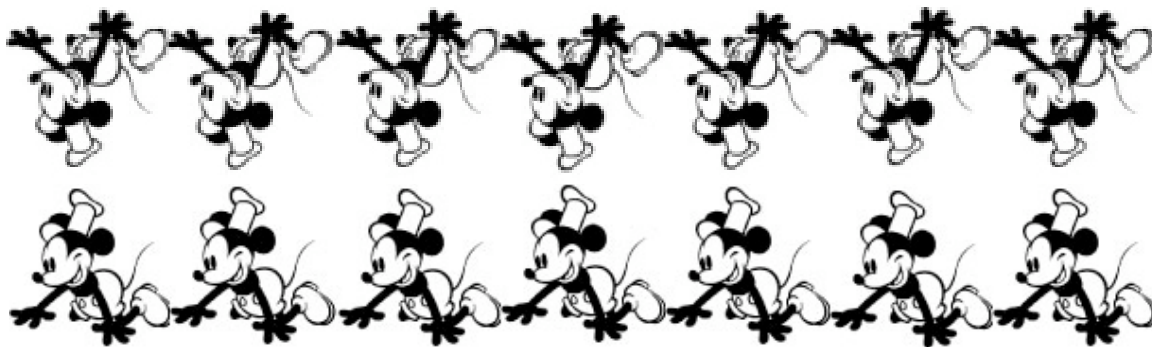
3. List (without justification) all of the symmetries of the objects *A* and *B* below.

(Describe clearly the symmetries that you list. For example, if you list a rotation as a symmetry then say “a rotation centered at *O* and for an angle of *x* many degrees”, where *O* is a point you should identify in the picture, and *x* is a specific angle measure you should find.)

(a)[5.5] Object A



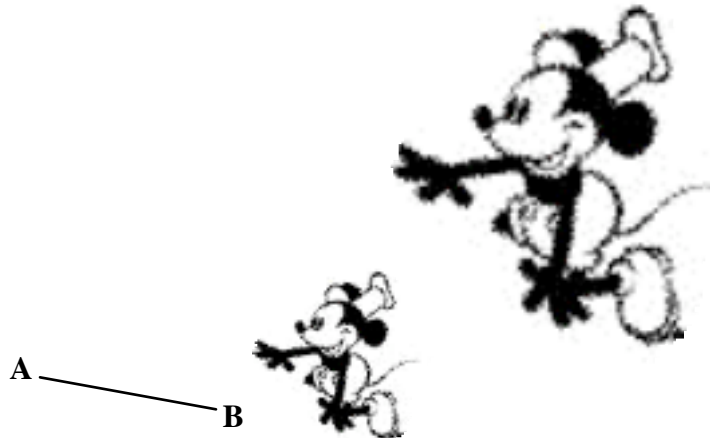
(b)[7.5] Object B (Assume that this pattern extends unboundedly to the left and to the right. Ignore minor details in the shapes and positions of the copies of Mickey.)



4. The larger Mickey (see the picture below) is obtained from the smaller Mickey by applying to the latter a central similarity  $f$ .

(a)[5.5] Construct the center of the similarity  $f$ .

(b)[7.5] Construct the image of the line segment AB (see the picture below) under the similarity  $f$ . You may refer to any Euclidean construction done in class without getting into details.



5. [BONUS 5] True or false? Do **not** justify anything in this question.

(a) It is possible to subdivide any given angle into three smaller equal angles by means of a Euclidean construction (with an unmarked ruler and a compass).

(b) In an acute golden triangle the ratio of the length of the base by the length of one of the other two sides is the golden ratio.

(c) For any three consecutive Fibonacci number the largest of them is the sum of the other two.

(d) The composition of the rotation around a point  $O$  by  $45^\circ$  followed by the rotation around the same point by  $55^\circ$  is the rotation around  $O$  by  $100^\circ$ .

(e) Any central similarity with stretching factor equal to 1 is the same as the identity symmetry.

This midterm exam is worth 25% of your final mark. There are 50 points distributed into the first 4 questions, and 5 BONUS points for question #5.