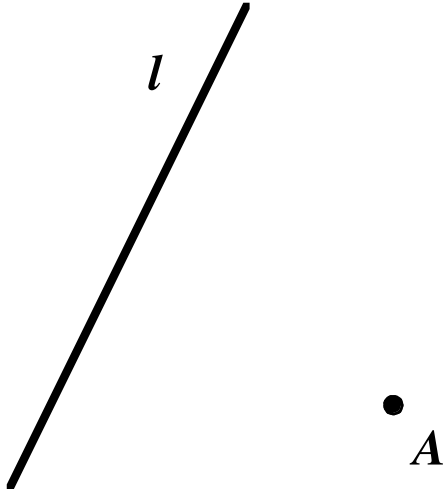


**136.102 Math in Art
Midterm Exam
February 21, 2006
Solutions**

Important: “Construct” means “construct using an unmarked ruler and a compass”. The phrase “unmarked ruler” stands for any ruler that may be used only as a straight edge to draw straight line segments. When you use a compass, show the (intermediate) circular arcs you draw in your constructions (do not erase them). Use words to describe BRIEFLY what you have done.

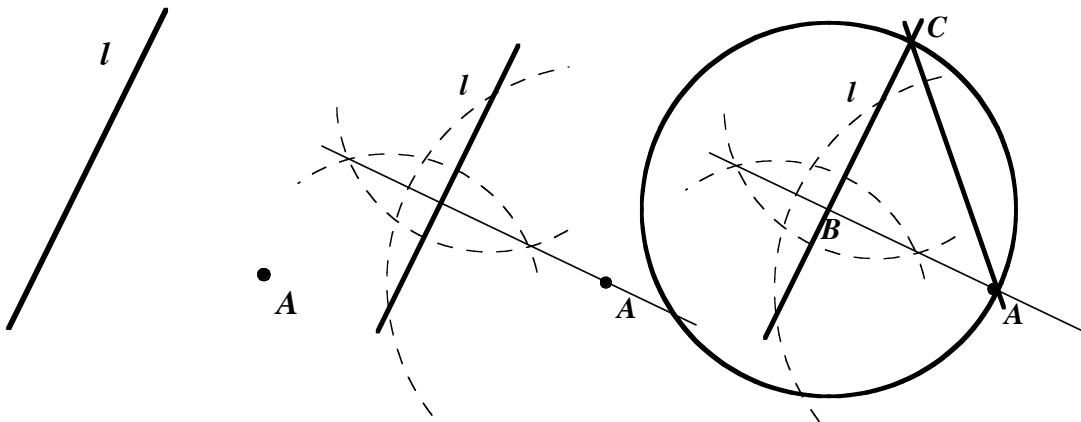
[10] 1.(a) Construct the line passing through the point A and parallel to the given line l .



Solution. This has been done in class, and it is also in the textbook. A solution is also posted on the web page. (5 points)

(b) Construct a line passing through the given point A (in the illustration shown below) and intersecting the given line l at the angle of 45° .

Solution. A solution is given at the right-hand side pictures: first construct a line through A that is perpendicular to l (middle picture), then a circle centered at B and with radius AB and finally join A and C (the right-hand side illustration.) (5 points.)



[10] 2. (a) Construct a regular pentagon over the given line segment (as one of the sides of the pentagon).

Solution. In the book and in the class notes. (6 points.)



(b) Construct an angle of 144° .

Solution. The solution of part (a) involves a construction of an acute golden triangle. The angles at the base of that triangle are both 72° . So, in order to get an angle of 144° we need to duplicate the angle of 72° . The duplicating of angle construction was done in class (it is also in the book). **(4 points.)**

[8] 3. (a) What are Fibonacci numbers? (Write down the definition.)


An infinite sequence of numbers $f_1, f_2, \dots, f_n, \dots$ defined by $f_1 = 1 = f_2$ and $f_{n+2} = f_{n+1} + f_n$ for $n \geq 1$. **(2 points.)**


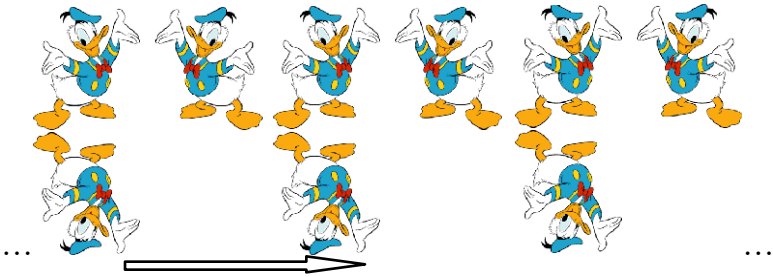
(b) It can be verified that $f_{25} = f_{24} + 28657$ and that $f_{22} = 17711$ (no need to check that). Find f_{24} .

(In this question, as usual, f_n is the n -th Fibonacci number in the list of Fibonacci numbers.)

Solution. Since $f_{25} = f_{24} + 28657$ and since $f_{25} = f_{24} + f_{23}$ it follows that $f_{23} = 28657$. Since $f_{24} = f_{23} + f_{22}$ we have that $f_{24} = 28657 + 17711 = 46369$. **(6 points.)**

[12] 4. Find the group of symmetries for each of the three objects shown below. If you claim a rotational symmetry, indicate the center of the rotation and the angle of rotation. If there are reflections, show the line of reflection. If there are translational symmetries show or describe the vectors of translation.

OBJECT	THE GROUP OF SYMMETRIES
	<p>{identity, reflection with respect to the vertical line passing through the center of the graphics} (3 points.)</p>

	<p>{identity, reflection with respect to the horizontal line as shown, reflection with respect to the vertical line as shown, rotation 180° centered at the intersection of the two lines that are shown} (4 points.)</p>
 <p>[This is a Frieze pattern (the pattern with ducks extends without end both to the left and to the right).]</p>	<p>Denote the vector shown to the left by v.</p> <p>{identity, translation(v), translation($2v$), translation($3v$),..., translation($-v$), translation($-2v$), translation($-3v$),...}</p> <p>(infinitely many symmetries) (5 points.)</p>

[10] 5. Suppose the point $f(A)$ is the image of the point A and the point $f(B)$ is the image of the point B under the central symmetry f . Find (construct) the center of the central symmetry f and then construct the image $f(C)$ of the point C (as shown in the illustration) under the central symmetry f .

Solution. First find the center of the similarity O as shown in the first picture below. [4 points.] Then join OC , join CB and finally **construct** (not shown in this brief solution) the line through $f(B)$ that is parallel to CB as shown in the second picture below. [6 points.]

