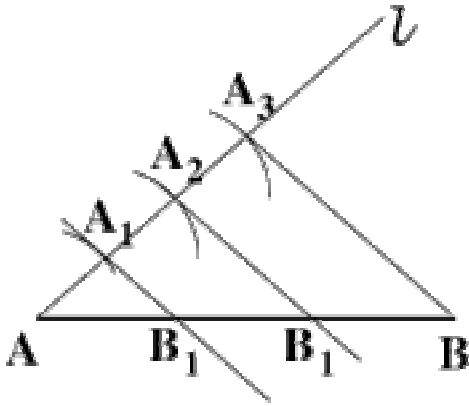


**136.102 Math in Art**  
**Midterm Exam**  
**February 24, 2005**

**Brief solutions**

1. Subdivide the given line-segment into three equal parts (using an unmarked ruler and a compass). Do not forget to briefly describe your steps.



We are given the line segment  $AB$ . First draw any line through  $A$  that is not passing through  $B$ . In the picture it is denoted by  $l$ . Then mark on  $l$  three equidistant points  $A_1$ ,  $A_2$  and  $A_3$  (use a compass, any radius). Join  $A_3$  and  $B$ . Then **CONSTRUCT** two lines passing through  $A_1$  and  $A_2$  respectively and that are parallel to the line through  $A_3$  and  $B$  (the construction is not shown here but should have been shown in the exam; we did it in class). The intersection points with the line segment  $AB$  will subdivide it into three equal parts.

2. (a)[6] Construct a golden rectangle to the right of the line segment given below (the **shorter** side of the rectangle). Do not forget to **briefly** describe your steps.

This is completely solved in the textbook (Section 3 in Chapter 1).

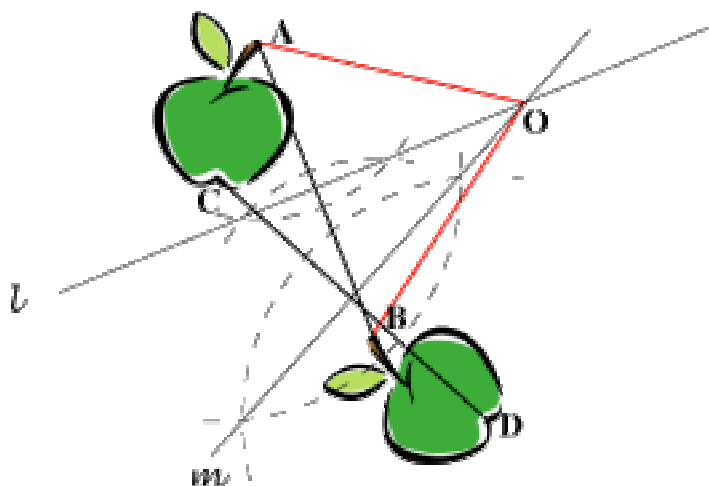
3. (a) What are Fibonacci numbers? (Write down the definition.)

It is a sequence of numbers  $f_1, f_2, f_3, \dots, f_n, \dots$  such that  $f_1 = 1$ ,  $f_2 = 1$  and any other number in that sequence is the sum of the preceding two numbers.

(b) The 22<sup>nd</sup> Fibonacci number  $f_{22}$  is 17711. The 20<sup>th</sup> Fibonacci number  $f_{20}$  is 6765. Find the 21<sup>st</sup> Fibonacci number  $f_{21}$ .

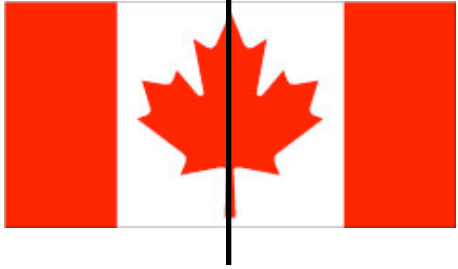
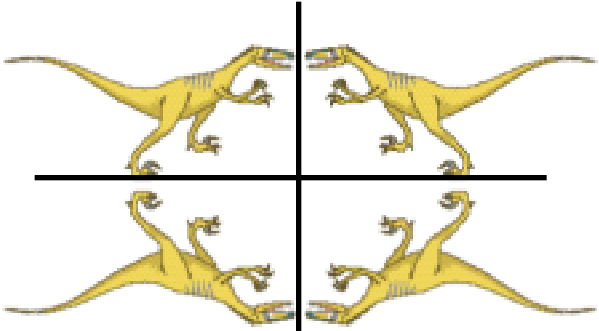
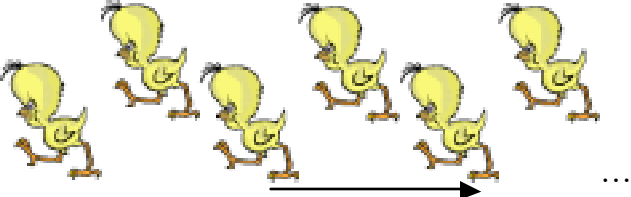
Since  $f_{22} = f_{21} + f_{20}$ , it follows that  $f_{21} = f_{22} - f_{20}$ , so that  $f_{21} = 17711 - 6765 = 10946$ .

4. The lower image of an apple is obtained by rotating the other image of the apple around a center  $O$  and by an angle  $\theta$ . Construct the center  $O$  and identify the angle of rotation  $\theta$ .



Bisect the line segment  $AB$  (connecting two corresponding points on the apples). That gives the line  $l$ . Then bisect any other line segment connecting two corresponding points on the two apples (in the pictures, these are the points  $C$  and  $D$ ). Get the line  $m$ . The center  $O$  of the rotation is in the intersection of the lines  $m$  and  $l$ . The angle of the rotation is  $\angle AOB$  (in red).

5. Find the group of symmetries for each of the three objects shown below. If you claim a rotational symmetry, indicate the center of the rotation and the angle of rotation. If there are reflections, show the line of reflection. If there are translational symmetries show or describe the vectors of translation.

OBJECT	THE GROUP OF SYMMETRIES
	<p>Identity and the reflection with respect to the vertical line shown in the picture.</p>
	<p>identity, reflection with respect to the horizontal line, reflection with respect to the vertical line and rotation through <math>180^\circ</math> around the intersection of the vertical and the horizontal line.</p>
 <p>[This is a Frieze pattern (the pattern with chickens extends without bounds both to the left and to the right). ]</p>	<p>identity, translation(<math>v</math>), translation(<math>2v</math>), ..., translation(<math>-v</math>), translation(<math>-2v</math>),...</p> <p>where <math>v</math> is the vector shown in the picture. (Note that <math>v</math> has a precise length and that precise length should be clearly visible.)</p>

