136.272

Assignment 3 (Sections 15.3-15.7)

Handed: Nov.16 2003. **Due: Nov.23 2003** in class. Show your work; providing answers without justifying them would not be sufficient.

1. [4 marks].

(a) [2] Find $f_x(0,0)$ and find $f_y(x,y)$ if $f(x,y) = e^{xy}\sin(x+y+\pi)$.

(b) [2] Find all (four) second order partial derivatives of $g(x,y) = xy^2 + \ln(x+y)$.

2. [5 marks]

(a) [1.5] Find the directional derivative of the function $f(x,y) = \dfrac{x^2 - y^2}{x^2 + y^2}$ in the direction of the vector $\left(\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right)$ at the point (1,-2).

(b) [2] Find the directions and the values of the smallest and the largest directional derivatives of the function $f(x,y) = \dfrac{x^2 - y^2}{x^2 + y^2}$ at the point (1,-2).

(c) [1.5] If $z^3 - xy + yz + y^3 - 2 = 0$ defines z as a function on x and y, find $\dfrac{\partial z}{\partial x}$ at the point (1,1,1).

3. [5 marks] First locate the local extrema of the function $g(x,y) = \dfrac{x+y}{x^2 + y^2 + 8}$, and then use the second derivative test to classify these local extrema (as local minima, local maxima or neither).

4. [6 marks] Consider the function $f(x,y) = x^2 - x - y + y^2$ over the points in the closed disk bounded by the circle $x = 2\cos t$, $y = 2\sin t$. Find and classify the **absolute** extrema of the function $f(x,y)$ over the (above) given domain.

5. [5 marks]. Find the dimensions of the rectangular box with no top and volume of 12 cubic meters, which has the smallest surface area.