

136.272

Assignment 2 (Sections 14.3, 14.4, 15.1-15.2)

Posted: Oct.17 2005; handed Oct. 19, 2005. Due: Oct.24 2005 in class. (If you hand it in by Friday, Oct 21, you will get it back before the midterm.) Late assignments will not be accepted.

Show your work; providing answers without justifying them would not be sufficient.

1. [9 marks] A spiral curve is defined by the vector function $\vec{r}(t) = (4 \cos t, 4 \sin t, 3t)$.
- (a) Find the arc length function $s(t)$ measured from the point $(4,0,0)$.
 - (b) Reparametrize the curve in terms of the arc length function s measured from the point $(4,0,0)$.
 - (c) Compute the curvature of that spiral curve at any moment in terms of s .
 - (d) Compute the curvature in terms of t directly from $\vec{r}(t) = (4 \cos t, 4 \sin t, 3t)$.
 - (e) Find the equations of the normal and the osculating plane to the spiral at the point $(4,0,0)$.

2. [4 marks] Find the position vector $\mathbf{r}(t)$ of the particle with acceleration $\mathbf{a}(t) = (\sin t, \cos t, 1)$, the initial velocity $\mathbf{v}(0) = (0,0,0)$ and initial position $\mathbf{r}(0) = (0,0,2)$. Where is the particle at the moment when $t = \pi$?

3. [6 marks] Determine **and sketch** (in the xy -plane) the domain of each of the following functions.

(a) $f(x, y) = \sqrt{x + y^3}$

(b) $g(x, y) = \frac{x + y}{1 - \sqrt{xy}}$

- [8] 4. [6 marks]

(a) Find the limit or show it does not exist: $\lim_{(x,y) \rightarrow (0,0)} \frac{1 - \cos(x^2 + y^2)}{x^2 + y^2}$

(b) Find the limit or show it does not exist: $\lim_{(x,y) \rightarrow (1,-1)} \frac{x^2 - 1}{x^2 + y^3}$