

136.272

Assignment 1 (Sections 13.1-13.7, 14.1-14.2)

Handed: Oct. 3, 2005. Due: Oct. 10, 2005 in class

Show your work (of course). Providing answers without justifying them will not be sufficient.

1. [5 marks] Find the equation of the plane which contains the point $(3, -1, 5)$ and is perpendicular to the intersection of the planes $x - 5y + 2z = 3$ and $4x + y - z = 2$.

2. [4 marks] Sketch the surface $x^2 + z^2 = -y$. Sketch at least two traces of that surface with planes of type $y = c$ (c various constants) and at least two traces with planes of type $z = c$ (c various constant).

3. [8 marks]

[1.5] (a) Find the rectangular coordinates of the point $(r, \theta, z) = \left(1, \frac{\pi}{4}, 1\right)$ given in cylindrical coordinates.

[1.5] (b) Find the rectangular coordinates of the point $(\rho, \theta, \phi) = \left(\frac{1}{2}, \frac{3\pi}{2}, \frac{\pi}{4}\right)$ given in spherical coordinates.

[2] (c) Find the cylindrical coordinates of the point $(x, y, z) = (3, -3, 4)$ given in rectangular coordinates.

[2] (d) Find the spherical coordinates of the point $(x, y, z) = (0, 6, 8)$ given in rectangular coordinates. Do not simplify here (specifically, instead of finding ϕ , find, say, $\cos \phi$).

[1] Plot all of the above points in a single coordinate system.

4. [8 marks].

[3] (a) Find all points on the curve $\vec{r}(t) = (3t^6, 4t^2 - 1, t^4 - 8t^2)$ where that curve has horizontal tangent lines.

[3] (b) Suppose l is any line perpendicular to the plane $x + y + z = 2$. Find the angle between that line and the tangent vector to the curve $\vec{r}(t) = (t^2 - 1, t^3 + t, 3 + t^2)$ when $t = -1$.

[2] (c) Find the unit tangent vector of the curve $x = t, z = t^2$ at the point $(2, 4)$.