136.271

Assignment 3: Sections 9.5 (the part on differentiation and integration of series), 9.6 and 9.7 and 9.8

(Due March 21 in class)

Note: show your work; a naked final answer is not worth anything.

1. [6 marks] Find the sums of the following series.

(a) $\sum_{n=2}^{\infty} n(n-1)x^n$, $|x|<1$.

(b) $\sum_{n=2}^{\infty} \frac{n^2-n}{2^n}$.

2. [5 marks] Find the Maclaurin series representation of the following functions.

(a) e^{3x}

(b) $\sin^2 x$

3. [6 marks] Find the sum of the series.

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n}(2n)!}$

(b) $\sum_{n=2}^{\infty} \frac{x^{3n+1}}{n!}$

(c) $\sum_{n=0}^{\infty} \frac{x^n}{2^n(n+1)!}$

4. [4 marks] Evaluate the following integrals as power series.

(a) $\int_0^x \sin(t^2)\,dt$

(b) $\int_0^x e^{t^3}\,dt$

5. [4 marks] Show that the Lagrange remainder in the Taylor's formula for the following functions tends to 0 as n tends to infinity, thus establishing that the functions are equal to their power series representations.

(a) $\cos 4x$

(b) e^{-2x}