# 136.271

# Assignment 1 (Sections 9.1 and 9.2)

**1.** [6 marks] Show that $\lim_{n\to\infty} \frac{3n-1}{4n+2} = \frac{3}{4}$ by using the definition of a convergent sequence and no other properties of sequences.

**2.** [7 marks] Consider the sequence $\{a_n\}$ defined by $a_1 = \sqrt{2}$ and $a_n = \sqrt{2a_{n-1}}$, $n > 1$.

(a) Compute $a_5$.

(b) Use mathematical induction to show that the sequence $\{a_n\}$ is bounded from above (Hint: show that $a_n < 10$, say.)

(c) (Optional) Use mathematical induction to show that the sequence $\{a_n\}$ increases.

(d) Use (b) and (c) above and refer to a theorem given in class (and in the textbook) to conclude that $\{a_n\}$ converges.

(e) Find $\lim_{n\to\infty} a_n$ (Hint: see how we have done that part in the similar examples done in class.)

**3.** [6 marks] Use what was covered in section 9.1 to evaluate the following limits.

(a) $\lim_{n\to\infty} \frac{\sqrt[3]{n}}{\sqrt{n}+1}$

(b) $\lim_{n\to\infty} \frac{1+2^n}{e^n}$

**4.** [6 marks] Find the sum if the series converges; otherwise show it diverges.

(a) $\sum_{n=1}^{\infty} \left( \frac{1}{2^{n-1}} + \frac{2}{3^{n-1}} \right)$

(b) $\sum_{n=1}^{\infty} \frac{n}{\sqrt{1+n^2}}$

(c) $\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$