

## 136.271

### Assignment 3: Section 9.8 and Uniform Convergence

(Due April 7 in class)

**Note: show your work; a naked final answer is not worth anything.**

1.

(a) Use the binomial series to expand  $x(1-x)^{-2}$ . Simplify your answer.

(b) Use part (a) to find the sum of the series  $\sum_{n=1}^{\infty} \frac{n}{2^n}$ . (No marks if other methods are used.)

2. Given the sequence of functions  $\{f_n(x)\}$  find the pointwise limit  $f(x)$  and then show that the sequence  $\{f_n(x)\}$  converges uniformly to  $f(x)$ .

(a)  $f_n(x) = \frac{n+x}{n}$  over the interval  $[0,1]$ .

(b)  $f_n(x) = \frac{\ln(1+nx)}{n}$  over the interval  $[1,2]$ .

3. Given the sequence of functions  $\{f_n(x)\}$  find the pointwise limit  $f(x)$  and then show that the sequence  $\{f_n(x)\}$  does **NOT** converge uniformly to  $f(x)$ .

(a)  $f_n(x) = \frac{n+x}{n}$  over the interval  $[0,\infty)$

(b)  $f_n(x) = \frac{n}{e^{nx^2}}$  over the interval  $[0,1]$ .