136.271 Assignment 1

Due January 28, 2004, in class

1. Use **only the definition** of the limit of a sequence to show that $\lim_{n\to\infty} \frac{1-n}{2n-2} + 2 = \frac{3}{2}$.

2. Consider the sequence $\{a_n\}$ defined by $a_1 = 1,\ a_{n+1} = \frac{1+2a_n}{1+a_n}$, n=1,2,3,….

   (a) Write down the first 5 members of that sequence.
   (b) Use induction to show that the sequence is bounded.
   (c) Use induction to show that the sequence increases.
   (d) Find the limit of that sequence.

3. Which of the following sequences converge, which diverge? If a sequence converges find the limit. (You may use the properties and theorems we have stated in class.)

   (a) $a_n = 1 + (-1)^n$

   (b) $a_n = \left(\frac{n+1}{2n}\right)\left(1 - \frac{1}{n}\right)$

   (c) $a_n = \frac{\ln(n+1)}{\sqrt{n}}$

   (d) $a_n = \left(\frac{1}{3}\right)^n + \frac{1}{\sqrt{2^n}}$

4. Which of the following series converge, which diverge? If a series converges, find its sum, and if a series diverges give reasons.

   (a) $\sum_{n=0}^{\infty} \frac{2^{n+1}}{5^n}$

   (b) $\sum_{n=0}^{\infty} \frac{2^{2n}}{3^n}$

   (c) $\sum_{n=1}^{\infty} \frac{6}{(2n-1)(2n+1)}$

   (d) $\sum_{n=0}^{\infty} \frac{n!}{1000^n}$