

136.270

Assignment 3 (Sections 14.4, 14.5, 14.6)

Handed: November 8, 2004. Due: November 15, 2004 in class.

Show your work. Providing answers without justifying them will not be sufficient.

1. [5 marks] [2](a) Find $\frac{\partial w}{\partial v}$ when $u = 1$ and $v = 2$ if $w = xy + \ln z$, $x = \frac{v^2}{u}$, $y = u + v$ and $z = \cos u$.

[3] (b) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at the point $(1,1,1)$ if z is the function on x and y defined by the equation $z^3 - xy + yz = 2 - y^3$.

2. [7 marks] [4] (a) Find the point on the surface $z = x^3y + y$ where the tangent plane is parallel to the plane $-3x - 2y + z = 1$. Then find the equation of the tangent plane at that point.

[3] (b) Find the equation of the normal line to the surface $z = x^3y + y$ at the point $(1,2,4)$.

3. [6 marks] At 10:00 a.m. a plane traveling east is 10 kilometers above a southbound car. The plane travels horizontally at 500 km/hour while the car maintains a constant speed of 100 km/hour. How fast is the distance between the plane and the car increasing at 11:00 a.m.? Do NOT simplify your answer.

4. [7 marks][3.5] (a) Find the directional derivative of the function $f(x,y,z) = xy + yz + zx$ at the point $(1,-1,2)$ in the direction of the vector $(3,6,-2)$.

[3.5] (b) Find the direction in which the function $f(x,y,z) = xy + yz + zx$ increases the most rapidly at the point $(1,-1,2)$. In which direction the same function decreases the most rapidly at $(1,-1,2)$? Find the maximal rate of change of $f(x,y,z) = xy + yz + zx$ at the point $(1,-1,2)$.