PAPER NO: 269 PAGE 1 OF 7

DEPARTMENT & COURSE NO: 136.270 TIME: 2 HOURS

EXAMINATION: Calculus 3A EXAMINERS: V. Charette

Values

[12] 1. Find an equation of the tangent plane to the graph of $f(x,y) = y^2\sqrt{x}$ at $(4,2,8)$.

[6] 2. (a) Suppose $F(x,y,z)$ has continuous partial derivatives. Find $\frac{\partial F}{\partial u}\left(3uv, \sin v, u^2\right)$ in terms of the partial derivatives of F.

[6] (b) Suppose $G(x,y) = xe^y$, where $x = x(t)$, $y = y(t)$ satisfy $x(0) = -1$, $y(0) = 1$, $x'(0) = 2$ and $y'(0) = 1$. Find $\frac{d}{dt}G(x(t), y(t))$ at $t = 0$.

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[12] 10. Compute the line integral $\int_C \left(xy + e^{\sqrt{x}}\right)dx + \left(x^2 + \cos\left(y^2\right)\right)dy$ where C is the positively oriented curve bounding the region enclosed by the parabolas $y = x^2$ and $x = y^2$.

Values

[12] 9. Let C be the curve from $(0,0,1)$ to $(1,1,4)$ in the intersection of the surfaces $x=y^2$ and $z=2x-y+1$.

Evaluate the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x,y,z)=-y\mathbf{i}+z\mathbf{j}+\mathbf{k}$.

Values

[12] 8. Compute the line integral $\int_C (x+2y)\,ds$, where C is the segment of the helix

$$\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + t\,\mathbf{k}, \quad 0 \le t \le \frac{\pi}{2}.$$

Values

[12] 6. Find the volume of the solid under the graph of $z=\frac{1}{\sqrt{1+x^2+y^2}}$ and above the region in the first quadrant bounded by $x^2+y^2=4$, $y=0$, $y=x$.

[6] 7. Using an iterated integral, find a formula for the volume of the solid contained in the first octant $(x\geq 0, y\geq 0, z\geq 0)$, bounded by the planes $x+y+z=2$ and $-2x-y+z=0$. (Do not evaluate the integral).

Values

[12] 4. Find and classify all the critical points of $f(x,y)=\frac{x^3}{3}-x^2-xy-\frac{y^2}{2}$.

[12] 5. Compute the double integral $\int_0^4 \int_{-2y}^{7y} \sqrt{x+2y}\, dx\, dy$.

Values

3. Let $f(x,y) = xy$ and let $\mathbf{u} = \frac{1}{\sqrt{2}}\mathbf{i} - \frac{1}{\sqrt{2}}\mathbf{j}$.

[4] (a) Find $\nabla f(-1,1)$, the gradient of f at $(-1,1)$.

[4] (b) At the point $(-1,1)$, find $D_{\mathbf{u}}f(-1,1)$, the directional derivative of f in the direction of **u**.

[4] (c) In what direction does f **decrease** most rapidly at the point $(-1,1)$?

[6] (d) Sketch the plane vector field $\nabla f(x,y)$, plotting the vectors at the points A, B, C, D indicated below.