THE UNIVERSITY OF MANITOBA

DATE: Oct. 22, 2001 MIDTERM EXAMINATION

DEPARTMENT & COURSE NO: 136.270 TIME: 1 hour

EXAMINATION: Calculus 3A EXAMINER: Dr. F. Ghahramani

NAME: (PRINT)______________________________

STUDENT NUMBER:______________________________

SIGNATURE:______________________________
(I understand that cheating is a serious offense)

INSTRUCTIONS TO THE STUDENT

This is a one-hour exam. There are 3 pages of questions and one blank page for rough work. Check now that you have all 3 pages of questions.. Answer all the questions in the spaces provided. If necessary, you may continue your work on the reverse sides of the pages but PLEASE INDICATE CLEARLY that your work continues and where the continuation may be found. DO NOT detach any question pages from the exam.

The point value of each question is indicated to the left of the question number. The maximum score possible is 60 points.

Please present your work CLEARLY and LEGIBLY, and use a pen (not red) or a dark pencil. **Justify your answers unless otherwise stated.**

NO CALCULATORS, TEXTS, NOTES OR OTHER AIDS ARE PERMITTED.

Values

1. For the curve C with the equation $\mathbf{r}(t) = \frac{2}{3}t^3\mathbf{i} + t^2\mathbf{j} + t\mathbf{k}$,

[5] (a) Find the length of the arc between the points corresponding to $t = 0$ and $t = 1$.

Note: $4t^4 + 4t^2 + 1 = \left(2t^2 + 1\right)^2$.

[5] (b) Find the unit tangent vector $\vec{T}$ at a general point. Simplify as much as possible.

[5] (c) Find the unit principal normal vector $\vec{N}$ at a general point. DO NOT SIMPLIFY.

Values

[8] 2. Show that the following limit does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{xy + x^2}{x^2 + y^2 + xy}$$

[13] 3. For the surface S given by the equation $z = x^2 - 3y^2 + xy$, find the equations of the tangent plane and the normal line at the point $(1,1,-1)$ on S.

Values

[12] 4. Let $f(x,y,z) = x^3e^y + z^2y$, find $\dfrac{\partial^2 f}{\partial x \partial y}$ and $f_{3,2,2}(x,y,z)$.

[12] 5. Let $z = ye^x + xe^y$, $x = s + t$, and $y = s - t$. By using the chain rule, calculate $\dfrac{\partial z}{\partial t}$ and $\dfrac{\partial^2 z}{\partial s \partial t}$.