136.270

Assignment 3 (Sections 15.6-15.8)

Handed: Nov.5 2003. **Due: Nov.12 2003** in class. Show your work; providing answers without justifying them would not be sufficient.

1. [5 marks]

(a) Find the directional derivative of the function $f(x,y)=\dfrac{x^2-y^2}{x^2+y^2}$ in the direction of the vector $\left(\dfrac{1}{\sqrt{2}},\dfrac{1}{\sqrt{2}}\right)$ at the point (1,-2).

(b) Find the directions and the values of the smallest and the largest directional derivatives of the function $f(x,y)=\dfrac{x^2-y^2}{x^2+y^2}$ at the point (1,-2).

2. [5 marks] First locate the local extrema of the function $g(x,y)=\dfrac{x+y}{x^2+y^2+8}$, and then use the second derivative test to classify these local extrema (as local minima, local maxima or neither).

3. [5 marks] Consider the function $f(x,y)=x^2-x-y+y^2$ over the points in the closed disk bounded by the circle $x=2\cos t$, $y=2\sin t$. Find and classify the **absolute** extrema of the function $f(x,y)$ over the (above) given domain.

4. [5 marks]. A silo is constructed with cylindrical walls and a conical roof. Find the dimensions of such a silo (the radius of the cylinder, the height of the cylinder and the height of the conic roof) of volume 10 000 cubic meters which has the smallest surface area. (You may need to browse through some of your (old) books for formulas for volume and surface area of cylinders and cones.)

5. [5 marks] Use the method of Lagrange multipliers to find and classify the extrema of the function $f(x,y)=xy$ subject to the constraint $x^2+y^2-4=0$.