136.270

Assignment 2 (Sections 14.3, 15.1-15.3)

Handed: Oct.10 2003. Due: Oct.17 2003 in class. Show your work; providing answers without justifying them would not be sufficient.

1. [8 marks] A spiral curve is defined by the vector function $\vec{r}(t) = (4\cos t, 4\sin t, 3t)$.

(a) [1.5] Find the arc length function $s(t)$ measured from the point (4,0,0).

(b) [1.5] Reparametrize the curve in terms of the arc length function s measured from the point (4,0,0).

(c) [1.5] Compute the curvature of that spiral curve at any moment in terms of s.

(d) [1.5] Compute the curvature in terms of t directly from $\vec{r}(t) = (2\cos t, 2\sin t, 3t)$.

(e) [2] Find the equations of the normal and the osculating plane to the spiral at the point (4,0,0).

2. [4 marks] Determine **and sketch** (in the xy-plane) the domain of each of the following functions.

(a) [2] $f(x,y) = \sqrt{x + y^3}$

(b) [2] $g(x,y) = \dfrac{x+y}{1-\sqrt{xy}}$

3. [8 marks]

(a) [2.5] Find the limit or show it does not exist: $\displaystyle\lim_{(x,y)\to(0,0)} \frac{1-\cos(x^2+y^2)}{x^2+y^2}$

(b) [2.5] Find the limit or show it does not exist: $\displaystyle\lim_{(x,y)\to(1,-1)} \frac{x^2-1}{x^2+y^3}$

(c) [3] Can the function $f(x,y) = \dfrac{x^3}{x^2+y^2} + 1$ be defined at the point (0,0) so it becomes continuous? Do not forget to justify your answer.

4. [5 marks].

(a) [2] Find $f_x(0,0)$ and find $f_y(x,y)$ if $f(x,y) = e^{xy}\sin(x+y+\pi)$.

(b) [3] Find all (four) second order partial derivatives of $g(x,y) = xy^2 + \ln(x+y)$.