

136.270

Assignment 1 (Sections 13.1-13.7, 14.1-14.2)

Handed: Sept. 24 2003. Due: Sept. 31 2003 in class (and as we all know, for some strange reason Sept. 31 falls on Oct.1)

Show your work (of course). Providing answers without justifying them will not be sufficient.

1. [5 marks] Find the equation of the plane which contains the point $(3, -1, 5)$ and is perpendicular to the intersection of the planes $x - 5y + 2z = 3$ and $4x + y - z = 2$.

2. [4 marks] Sketch the surface $x^2 + z^2 = -y$. Sketch at least two traces of that surface with planes of type $y = c$ (c various constants) and at least two traces with planes of type $z = c$ (c various constant).

3. [8 marks]

[1.5] (a) Find the rectangular coordinates of the point $(r, \theta, z) = \left(1, \frac{\pi}{4}, 1\right)$ given in cylindrical coordinates.

[1.5] (b) Find the rectangular coordinates of the point $(\rho, \theta, \phi) = \left(\frac{1}{2}, \frac{3\pi}{2}, \frac{\pi}{4}\right)$ given in spherical coordinates.

[2] (c) Find the cylindrical coordinates of the point $(x, y, z) = (3, -3, 4)$ given in rectangular coordinates.

[2] (d) Find the spherical coordinates of the point $(x, y, z) = (0, 6, 8)$ given in rectangular coordinates. Do not simplify here (specifically, instead of finding ϕ , find, say, $\cos \phi$).

[1] Plot all of the above points in a single coordinate system.

4. [8 marks].

[3] (a) Find all points on the curve $\vec{r}(t) = (3t^6, 4t^2 - 1, t^4 - 8t^2)$ where that curve has horizontal tangent lines.

[3] (b) Suppose l is any line perpendicular to the plane $x + y + z = 2$. Find the angle between that line and the tangent vector to the curve $\vec{r}(t) = (t^2 - 1, t^3 + t, 3 + t^2)$ when $t = -1$.

[2] (c) Find the unit tangent vector of the curve $x = t$, $z = t^2$ at the point $(2, 4)$.