

136.170 Tutorial Worksheet 3

Fri Jan. 20, 2006 to Thurs Jan 26, 2006

1. Determine a region whose area is equal to the given limit. Do not evaluate the limit:

$$(a) \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{3}{n} \left(\frac{3i}{2n} + 3 \right)$$

$$(b) \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{\pi}{n} \sin \frac{i\pi}{n}$$

2. Express the limit as a definite integral on the given interval:

$$(a) \lim_{n \rightarrow \infty} \sum_{i=1}^n (x_i^2 - 3x_i) \Delta x, \quad [-7, 5]$$

$$(b) \lim_{n \rightarrow \infty} \sum_{i=1}^n \sqrt{4 - x_i^2} \Delta x, \quad [0, 1]$$

$$(c) \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{n} \cdot \frac{1}{2 + (i/n)^2}, \quad [0, 1]$$

3. Use area to evaluate the integrals:

$$(a) \int_{-2}^1 |x| dx$$

$$(b) \int_{-3}^3 \sqrt{9-x^2} dx$$

4. Suppose that f and g are continuous and

$$\int_1^2 f(x) dx = -4, \int_1^5 f(x) dx = 6, \int_1^5 g(x) dx = 8$$

Find

$$(a) \int_2^2 g(x) dx$$

$$(b) \int_5^1 g(x) dx$$

$$(c) \int_1^2 3f(x) dx$$

$$(d) \int_2^5 f(x) dx$$

$$(e) \int_1^5 [f(x) - g(x)] dx$$

$$(f) \int_1^5 [4f(x) - g(x)] dx$$