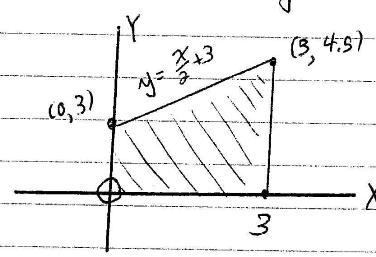
Math 170

Tutorial Worksheet 3

Solutions

1. (a) If $\Delta x = \frac{3}{n}$, then when $i=1$, $\frac{3i}{n} = \frac{3}{n}$ and when $i=n$, $\left(\frac{3}{n}\right)i = 3$. $\therefore$ the quantity $\frac{3i}{2n}+3$ evaluates the function $f(x)=\frac{x}{2}+3$ at the right-hand endpoint of the ith subinterval $\left[\frac{3(i-1)}{n}, \frac{3i}{n}\right]$ of $[0,3]$.

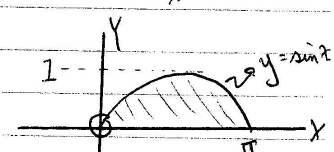
$\therefore \lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\left(\frac{3i}{2n}+3\right)$ is the area of the region bounded above by $f(x)=\frac{x}{2}+3$, below by $y=0$, to the left by $x=0$, and to the right by $x=3$



(b) When $i=1$, $\frac{i\pi}{n} = \frac{\pi}{n}$ and when $i=n$, $\frac{i\pi}{n} = \pi$. $\therefore \Delta x_i = \frac{\pi}{n}$

$\therefore$ the quantity $\sin\left(\frac{i\pi}{n}\right)$ evaluates the function $f(x)=\sin(x)$ at the right-hand endpoint of the ith subinterval $\left[\frac{(i-1)\pi}{n}, \frac{i\pi}{n}\right]$ of $[0,\pi]$.

$\therefore \lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi}{n} \sin\left(\frac{i\pi}{n}\right)$ is the area of the region bounded above by $y=\sin(x)$, below by the X-axis, to the left by $x=0$, and to the right by $x=\pi$



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2. (a) $\lim_{n\to\infty} \sum_{i=1}^{n} (x_i^2 - 3x_i)\Delta x_i$ on $[-7, 5]$

is $\int_{-7}^{5} (x^2 - 3x)\,dx$

(b) $\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{4 - x_i^2}\,\Delta x_i$ on $[0, 1]$ is $\int_0^1 \sqrt{4 - x^2}\,dx$

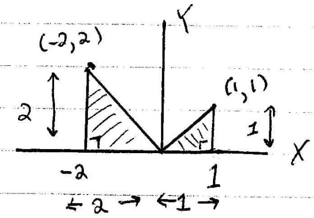
(c) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n} \frac{1}{2 + \left(\frac{i}{n}\right)^2}$ on $[0, 1]$ is $\int_0^1 \frac{1}{2 + x^2}\,dx$

3. (a) $\int_{-2}^{1} |x|\,dx$ = area of region below $y = |x|$, above the X-axis, and between $x = -2$ and $x = 1$

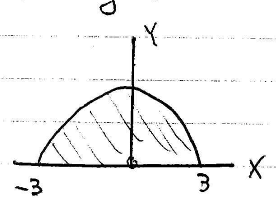
= sum of areas of the two shaded triangles

$= \left(\frac{1}{2}\right)(2)(2) + \left(\frac{1}{2}\right)(1, 1)$

$= 2 + \frac{1}{2} = 2.5$ square units.



(b) $y = \sqrt{9 - x^2}$ $(-3 \le x \le 3)$ is the equation of the semicircle with centre $(0,0)$ and radius 3 $(y \ge 0)$



$\therefore \int_{-3}^{3} \sqrt{9 - x^2}\,dx$ is the area of this semicircle, which is $\frac{1}{2}\pi(3^2) = \frac{9\pi}{2}$ sq. units

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4. We have $\int_1^2 f(x)\,dx = -4$, $\int_1^5 f(x)\,dx = 6$, and $\int_1^5 g(x)\,dx = 8$.

(a) $\int_2^2 g(x)\,dx = 0$ as the upper and lower limits of integration are equal.

(b) $\int_5^1 g(x)\,dx = -\int_1^5 g(x)\,dx = -8$

(c) $\int_1^2 3f(x)\,dx = 3\int_1^2 f(x)\,dx = 3\times(-4) = -12$

(d) $\int_2^5 f(x)\,dx = \int_1^5 f(x)\,dx - \int_1^2 f(x)\,dx = 6-(-4) = 10$

(e) $\int_1^5 [f(x)-g(x)]\,dx = \int_1^5 f(x)\,dx - \int_1^5 g(x)\,dx$

$= 6 - 8 = -2$

(f) $\int_1^5 [4f(x)-g(x)]\,dx = \left(4\int_1^5 f(x)\,dx\right) - \int_1^5 g(x)\,dx$

$= 4(6) - 8 = 24-8 = 16$

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