

Tutorial 1
Solutions

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Write it as a quotient with e^{-x} as denominator.
It then becomes an " $\frac{0}{0}$ indeterminate form" and we
can use L'Hopital's rule:

$$\lim_{x \rightarrow +\infty} (e^x) \left(\ln \left(1 + \frac{1}{x} \right) \right) = \lim_{x \rightarrow +\infty} \frac{\ln \left(1 + \frac{1}{x} \right)}{e^{-x}}$$

$$= \lim_{x \rightarrow +\infty} \frac{\left(\frac{1}{1 + \frac{1}{x}} \right) \left(-\frac{1}{x^2} \right)}{-e^{-x}} = \lim_{x \rightarrow +\infty} \frac{e^x}{x^2 + x} \quad \left(\text{an } \frac{\infty}{\infty} \right)$$

(after simplifying)

indeterminate form

$$= \lim_{x \rightarrow +\infty} \frac{e^x}{2x + 1} = \lim_{x \rightarrow +\infty} \frac{e^x}{2} = +\infty \quad \left(\text{after applying L'Hopital's rule 2 more times} \right)$$

$\frac{\infty}{\infty}$ form

(e) $\lim_{x \rightarrow +\infty} x e^{\frac{1}{x}} - x$ As $x \rightarrow +\infty$, $e^{\frac{1}{x}} \rightarrow e^0 = 1$, so this is an " $\infty - \infty$ " indeterminate form.

Rewrite it as a quotient: $\rightarrow \lim_{x \rightarrow +\infty} \frac{e^{\frac{1}{x}} - 1}{\frac{1}{x}}$

This is a $\frac{0}{0}$ indeterminate form

so we can use L'Hopital's rule and get:

$$\lim_{x \rightarrow +\infty} (x e^{\frac{1}{x}} - x) = \lim_{x \rightarrow +\infty} \frac{(e^{\frac{1}{x}}) \left(-\frac{1}{x^2} \right)}{\left(-\frac{1}{x^2} \right)}$$

$$= \lim_{x \rightarrow +\infty} e^{\frac{1}{x}} = e^0 = 1$$