Mathematics 170

Tutorial 1 Solutions

Wed. Jan. 12, 2005 - Tues. Jan. 18 2005

1.(a) $\lim_{x\to 1} \frac{x^{\frac{1}{3}}-1}{x^{\frac{1}{5}}-1}$ is an "$\frac{0}{0}$ indeterminate form", so by L'Hopital's rule we have: $\lim_{x\to 1} \frac{x^{\frac{1}{3}}-1}{x^{\frac{1}{5}}-1} = \lim_{x\to 1} \frac{\frac{1}{3}x^{-\frac{2}{3}}}{\frac{1}{5}x^{-\frac{4}{5}}}$

$= \lim_{x\to 1} \frac{5}{3} x^{\frac{2}{15}} = 5/3$.

(b) $\lim_{x\to+\infty} \frac{\ln(e^x+x)}{x}$ is a "$\frac{\infty}{\infty}$ indeterminate form," so by L'Hopital's rule we have: $\lim_{x\to+\infty} \frac{\ln(e^x+x)}{x} = \lim_{x\to+\infty} \frac{\left(\frac{1}{e^x+x}\right)(e^x+1)}{1}$

This is another $\frac{\infty}{\infty}$ indeterminate form, so use L'Hopital again and get

$\lim_{x\to+\infty} \frac{\ln(e^x+x)}{x} = \lim_{x\to+\infty} \frac{e^x}{e^x+1} = \lim_{x\to+\infty} \frac{1}{1+e^{-x}} = \frac{1}{1+0} = 1.$

(c) $\lim_{x\to 0^+} \frac{\sqrt{x}}{1-\cos x}$ ($\frac{0}{0}$ form) $= \lim_{x\to 0^+} \frac{\frac{1}{2\sqrt{x}}}{\sin x}$

$= \lim_{x\to 0^+} \frac{1}{2\sqrt{x}\sin x} = +\infty$.

(d) $\lim_{x\to+\infty} e^x\left(\ln\left(1+\frac{1}{x}\right)\right)$ is an "$\infty\cdot 0$" indeterminate form as $\lim_{x\to+\infty} \ln\left(1+\frac{1}{x}\right) = \ln(1)$
$= 0$.