SURNAME: (Print in ink) ______________________________

FIRST NAME: (Print in ink) ______________________________

STUDENT NUMBER: ______________________

SEAT NUMBER: ______________________

SIGNATURE: (Print in ink) ______________________________
(I understand that cheating is a serious offense)

Please indicate your instructor and section by checking the appropriate box below:

☐	L03	R.G. Woods	M,W,F	9:30 - 10:20
☐	L04	SK	M,W,F	11:30 - 12:20
☐	L05	Chandra	Tues, Thurs	1:00 - 2:20
☐	Sisler	☐ SJR	☐ Deferred Exam	☐ Challenge for credit

INSTRUCTIONS TO STUDENTS:

This is a 1 hour exam. **Please show your work clearly**.

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title pages, 5 pages of questions and also 2 blank pages for rough work. Please check that you have all the pages. You may remove the blank pages if you want, but be careful not to loosen the staples.

The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 40 points.

Answer all questions on the exam paper in the space provided bbeneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	7	
2	6	
3	6	
4	7	
5	7	
6	7	
Total:	40	

1. Evaluate the following integrals.

[3] (a) $\int \frac{2e^x}{e^x+2}\,dx$

With $u = e^x + 2$, $du = e^x dx$, we have:

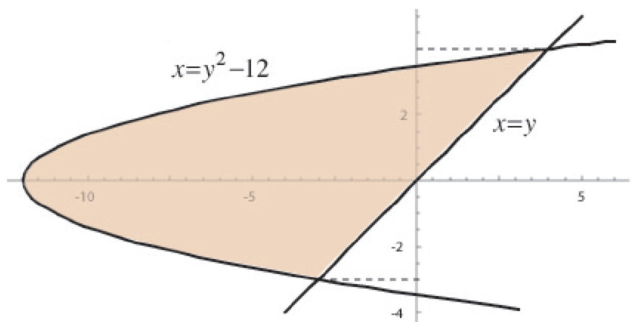
$$\int \frac{2e^x}{e^x+2}\,dx = \int \frac{2du}{u} = 2\ln|u| + c = 2\ln(e^x+2) + c$$

[4] (b) $\int_0^8 \frac{cos(\sqrt{x+1})}{\sqrt{x+1}}\,dx$

Use $u = \sqrt{x+1}$ so that $du = \frac{1}{2\sqrt{x+1}}dx$:

$$\int_0^8 \frac{cos(\sqrt{x+1})}{\sqrt{x+1}}\,dx = \int_{x=0}^{x=8} 2\cos u du = 2\sin u|_{x=0}^{x=8} = 2\sin(\sqrt{x+1})|_0^8 = 2\sin 3 - 2\sin 1$$

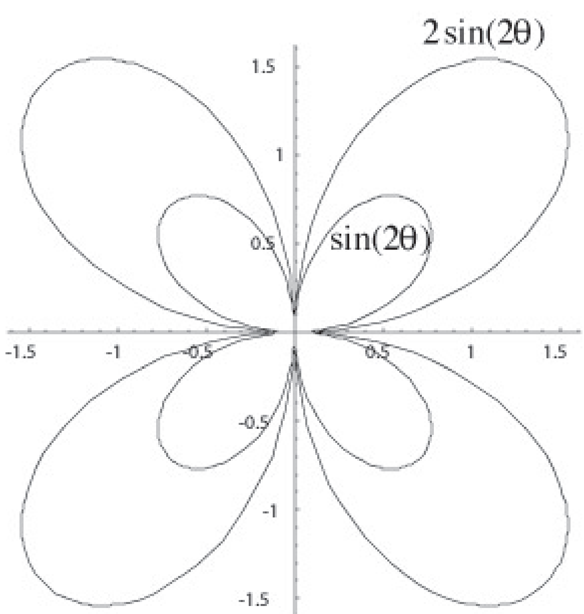
[6] 2. Find the area of the region bounded by the curves $x = y^2 - 12$ and $x = y$ as illustrated in the picture below.



Solving $x = y^2 - 12$, $x = y$ gives $y = -3$ and $y = 4$. So, the required area is:

$$\int_{-3}^{4} (y - (y^2 - 12)dy = \frac{y^2}{2} - \frac{y^3}{3} + 12y|_{-3}^{4} = 8 - \frac{64}{3} + 48 - \frac{9}{2} - 9 + 26.$$

[6] 3. Find the area of the region inside the curve $r = 2sin(2\theta)$ (given in polar coordinates) and outside the curve $r = sin(2\theta)$ (in polar coordinates), as illustrated in the picture below.



The required area is 4 times larger than the area of the region in the first quadrant. So we have:

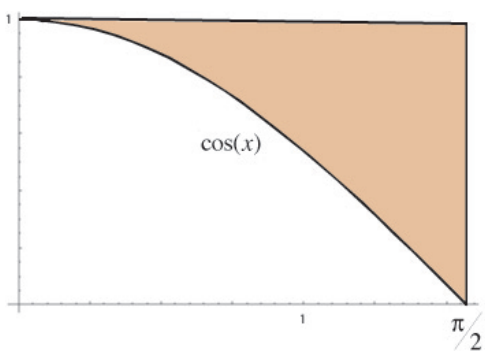
Now we use the identity $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$ applied both to $\sin^2\theta$ and to $\sin^2 2\theta$.

$$Area = 8\frac{1}{2}\int_0^{\frac{\pi}{2}}(1-\cos 4\theta)d\theta - \int_0^{\frac{\pi}{2}}(1-\cos 2\theta)d\theta =$$

$$4(\theta - \frac{1}{4}\sin 4\theta) - \theta + \frac{1}{2}\sin 2\theta|_0^{\frac{\pi}{2}} = 3\theta - \sin 4\theta + \frac{1}{2}\sin 2\theta|_0^{\frac{\pi}{2}} = 3\frac{\pi}{2}.$$

(In the following two question the graphs do not correspond precisely to what has been described in the text. That affects the limits of integration only. In the solutions below we give a priority to the text. Solutions where the graphs have been used as reference were also considered correct.)

[7] 4. The region R is bounded by the curves $y = cos(x)$, $y = 1$ and $x = 1$ as illustrated in the picture below. Find the volume of the solid obtained by revolving R around the x-axis. [Hint: washer method!]

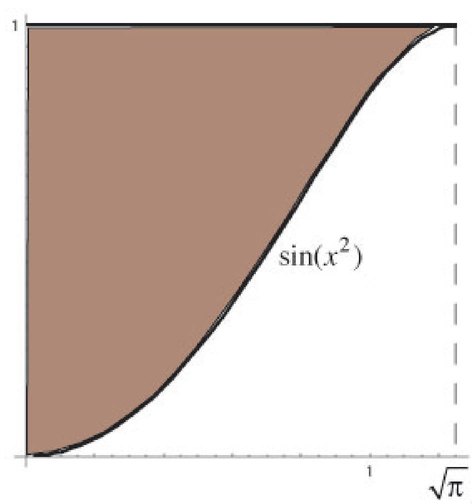


$$V = \int_0^1 (1^2 - \cos^2 x)\pi dx = \pi x||_0^1 - \pi\int_0^1 \cos^2 dx$$

We use the identity $\cos^2 x = \frac{1}{2}(\cos 2x + 1)$

$$V = \pi - \pi\int_0^1 \frac{1}{2}(\cos 2x + 1)dx = \pi - \pi\frac{1}{2}\frac{\sin 2x}{2} - \frac{\pi}{2}x|_0^1 = \pi - \frac{\pi \sin 2}{4} - \frac{\pi}{2}.$$

[7] 5. The region R is bounded by the curves $y = sin(x^2)$, $y = 1$ and $x = 0$ as illustrated in the picture below. Find the volume of the solid obtained by revolving R around the y-axis. [Hint: Cylindrical shells!]



$$V=\int_0^{\sqrt{\pi}} 2\pi x(1-\sin(x^2))dx = 2\pi\frac{x^2}{2}|_0^{\sqrt{\pi}} - 2\pi\int_0^{\sqrt{\pi}} x\sin(x^2)dx$$

We use the substitution $u=x^2$, so that $\frac{du}{2}=xdx$.

$$V=2\pi\frac{\pi}{2}-2\pi\int_{x=0}^{x=\sqrt{\pi}}\frac{1}{2}\sin u du = \pi^2+\pi\cos u|_{x=0}^{x=\sqrt{\pi}} =$$

$$=\pi^2+\pi\cos x^2|_0^{\sqrt{\pi}} = \pi^2-\pi-\pi=\pi^2-2\pi.$$

6. Evaluate the following integrals

[3] (a) $\int_0^{\frac{1}{2}} \frac{2+3x}{\sqrt{1-x^2}}\,dx$

$$I=\int_0^{\frac{1}{2}}\frac{2+3x}{\sqrt{1-x^2}}\,dx = \int_0^{\frac{1}{2}}\frac{2}{\sqrt{1-x^2}}\,dx + \int_0^{\frac{1}{2}}\frac{3x}{\sqrt{1-x^2}}\,dx$$

For the second integral we use $u=1-x^2$ so that $-2xdx=du$.

$$I=2\sin^{-1}x|_0^{\frac{1}{2}} + 3\int_{x=0}^{x=\frac{1}{2}}\frac{-du}{2\sqrt{u}} = 2\sin^{-1}\frac{1}{2}-2\sin^{-1}0-\frac{3}{2}\frac{u^{\frac{1}{2}}}{\frac{1}{2}}|_{x=0}^{x=\frac{1}{2}} =$$

$$=2\frac{\pi}{4}-3(1-x^2)|_0^{\frac{1}{2}} = \frac{\pi}{2}-3(\frac{3}{4})+3.$$

[4] (b) $\int 2x\ln(x)\,dx$

Use integration by parts: $u=\ln x$, $dv=xdx$ so that $du=\frac{dx}{x}$ and $v=\frac{x^2}{2}$.

$$\int 2x\ln(x)\,dx = 2\int x\ln(x)\,dx = 2\frac{x^2}{2}\ln x - 2\int\frac{x^2}{2}\frac{1}{x}dx = x^2\ln x-\frac{x^2}{2}+c.$$