THE UNIVERSITY OF MANITOBA

April 16, 2005	FINAL EXAMINATION
DEPARTMENT & COURSE NO: 136.170	PAGE 1 of 8
PAPER NO. 234	TIME: 2 Hours
EXAMINATION: Calculus 2	EXAMINERS: Various

Values

1. Calculate $\lim_{x\to+\infty} \frac{x\ln(x)}{x^2+1}$. Justify your calculations.

5

2. A curve C has parametric equations

$$y = 2t^3 - 6t$$
$$x = 2t^3 + 3t^2$$

Find the co-ordinates of the points on C at which the tangent line to C has slope $\frac{1}{2}$.

8

continued

3. Suppose that $g(x)$ is a differentiable function and that

$$\int_0^8 \left(\sqrt{x+1} + 3g'(x)\right)dx = 24$$

If $g(0) = \frac{7}{9}$, calculate the value of $g(8)$.

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4. Let C be a curve with polar equation $r = \sqrt{\sin(\theta)}$ for $0 \le \theta \le \pi$. Find the area of the region bounded by the positive y-axis, the curve C, and the line $\theta = \frac{2\pi}{3}$.

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5. Find the area of the portion of the first quadrant bounded between the curves $y = 8\sqrt{x}$ and $y = x^2$.

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6. Let R be the region of the first quadrant bounded below by the x-axis, above by the parabola $y = x^2$, and on the right by the vertical line $x = k$ (where $k > 0$). The volume of the solid swept out by rotating R around the x-axis equals the volume of the solid swept out by rotating R about the y-axis. What is the value of k? What is this common volume?

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7. Evaluate $\int_{-\frac{1}{2}}^{\frac{\pi-2}{4}} \sqrt{\sin(2x+1)}\cos(2x+1)dx$. Is your answer bigger than 0.25?

9

8. Evaluate the following integrals:

   (a) $\int \tan(2x)\sec^4(2x)dx$

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(b) $\int \frac{x^3}{\sqrt{9-x^2}}\,dx$

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(c) $\int \frac{2x^2-x+2}{x^3+2x}\,dx$

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9. Do the following improper integrals converge? If so, to what? Explain.

(a) $\int_0^{\infty} \frac{dx}{(2x+1)(x+1)}$

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(b) $\int_0^{1} \frac{e^x}{\sqrt{e^x - 1}} dx$

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10. The curve C has parametric equations

$$x = \frac{t^3}{3} + \frac{t^2}{2}$$

$$y = \frac{t^3}{3} - \frac{t^2}{2} \qquad (0 \le t \le 1).$$

What is the length of C?

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11. Let S be the surface swept out by rotating the curve $y = \frac{x^3}{3}$ $(0 \le x \le 1)$ about the x-axis. Compute the surface area of S.

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12. Evaluate $\lim_{x\to 0}\left(\dfrac{\int_{x}^{3x}\sin\left(t^2\right)dt}{\int_{0}^{2x}\sin\left(t^2\right)dt}\right)$.

9

120

THE END