B01.

MATH 1700: Test #5 (Fall 2011)

Solution; marking scheme

1. [8] Write out the form of the partial fraction decomposition of the function. Do not determine the numerical values of the coefficients.

[4] (a) $\dfrac{2}{\left(x^2+2x+2\right)\left(x^2+2x+1\right)}$

[4] (b) $\dfrac{2}{\left(x^2+2\right)^2\left(x^2+x-2\right)}$

Solution. (a) Since $x^2+2x+1=(x+1)^2$ and since x^2+2x+2 does not factor further, we should use $\dfrac{2}{\left(x^2+2x+2\right)\left(x^2+2x+1\right)}=\dfrac{Ax+B}{\left(x^2+2x+2\right)}+\dfrac{C}{(x+1)}+\dfrac{D}{(x+1)^2}$.

(b) x^2+2 does not factor, and $x^2+x-2=(x-1)(x+2)$. So, we should use

$$\frac{2}{\left(x^2+2\right)^2\left(x^2+x-2\right)}=\frac{A_1x+B_1}{x^2+2}+\frac{A_2x+B_2}{\left(x^2+2\right)^2}+\frac{C}{x-1}+\frac{D}{x+2}.$$

2. [11] Compute the integral, or show it diverges:

[5] (a) Compute $\displaystyle\int_1^2 \frac{1}{\sqrt{x-1}}\,dx$.

[6] (b) Does $\displaystyle\int_1^\infty \frac{1}{\sqrt{x^3+5x+1}}\,dx$ converge? Justify! (Hint: comparison test!)

Solution. (a) $\displaystyle\int_1^2 \frac{1}{\sqrt{x-1}}\,dx=\lim_{t\to1^+}\int_t^2 \frac{1}{\sqrt{x-1}}\,dx=\lim_{t\to1^+}\left(2\sqrt{x-1}\,\Big|_t^2\right)=\lim_{t\to1^+}(2-2\sqrt{t-1})=2$.

(b) Since $\sqrt{x^3+5x+1}>\sqrt{x^3}$ for $x\ge1$, we have $\dfrac{1}{\sqrt{x^3+5x+1}}<\dfrac{1}{x^{3/2}}$. Since $\displaystyle\int_1^\infty \frac{1}{x^{3/2}}\,dx$ converges, by the comparison test, so does $\displaystyle\int_1^\infty \frac{1}{\sqrt{x^3+5x+1}}\,dx$.

3. [6] Compute the length of the curve $y=\dfrac{2}{3}x^{3/2}$ for $0\le x\le1$.

Solution.

$$L=\int_0^1\sqrt{1+\left(\left(\frac{2}{3}x^{3/2}\right)'\right)^2}\,dx=\int_0^1\sqrt{1+\left(x^{1/2}\right)^2}\,dx=\int_0^1\sqrt{1+x}\,dx=\frac{2}{3}(1+x)^{3/2}\Big|_0^1=\frac{2}{3}2^{3/2}-\frac{2}{3}.$$