B04. **MATH 1700: Test #4 (Fall 2011)**

**Solution; marking scheme**

**1.** Evaluate the following integrals.

**[8] (a)** $\int_1^2 x^2 \ln\left(x^2\right) dx$

**[8] (b)** $\int \cos^2 x \sin^3 x \, dx$

**[8] (c)** $\int \frac{e^x}{\sqrt{4-e^{2x}}} dx$

*Solution*. (a) Integration by parts: $u = \ln\left(x^2\right)$, $dv = x^2 dx$; this gives $du = \frac{2}{x} dx$ and $v = \frac{x^3}{3}$. Hence

$$\int_1^2 x^2 \ln\left(x^2\right) dx = \frac{x^3}{3} \ln\left(x^2\right)\Big|_1^2 - \frac{2}{3}\int_1^2 x^2 \, dx = \frac{x^3}{3}\ln\left(x^2\right) - \frac{2}{3}\frac{x^3}{3}\Big|_1^2 = \frac{8}{3}\ln 4 - \frac{16}{9} + \frac{2}{9}.$$

(b) Use $u = \cos x$; this gives $du = -\sin x \, dx$, and we have

$$\int \cos^2 x \sin^3 x \, dx = \int \cos^2 x \sin^2 x \sin x \, dx = -\int u^2 \left(1-u^2\right) du = -\frac{u^3}{3} + \frac{u^5}{5} + c = -\frac{(\cos x)^3}{3} + \frac{(\cos x)^5}{5} + c$$

(c) First use $u = e^x$, so that $du = e^x dx$. We have $\int \frac{e^x}{\sqrt{4-e^{2x}}} dx = \int \frac{1}{\sqrt{4-u^2}} du$. Now use $u = 2t$; it gives $\int \frac{1}{\sqrt{4-u^2}} du = 2\int \frac{1}{\sqrt{4-4t^2}} dt = \sin^{-1} t + c = \sin^{-1}\frac{u}{2} + c = \sin^{-1}\frac{e^x}{2} + c$.

Give them 1point bonus for free, so that the total mark adds to 25.

NOTE: these are by no means the only solutions. Please mark carefully. Partial marks for partial solutions.